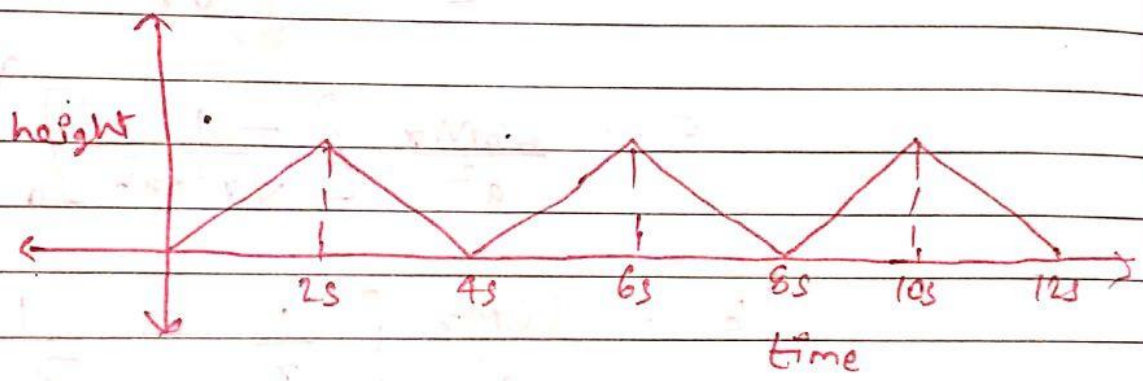


S. H. M

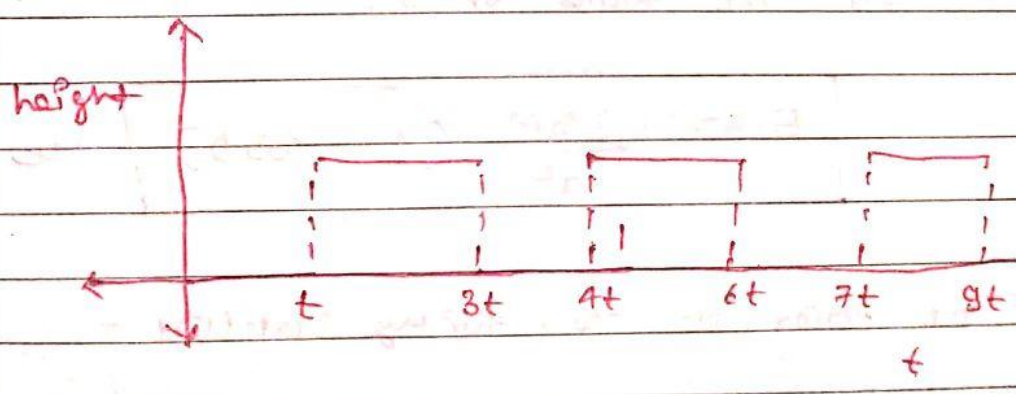
SHM is not a wave

• Periodic Motion : (Fixed) Regular Interval Repeat
Harmonic

① insect climbing a wall & falling down



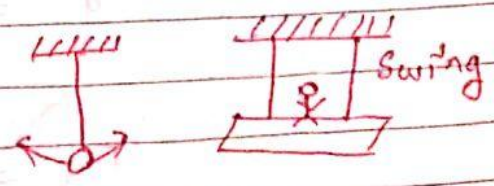
② Bacha → Steps → Climbs up & down



③ U.C.M

• Oscillatory : to & fro periodic

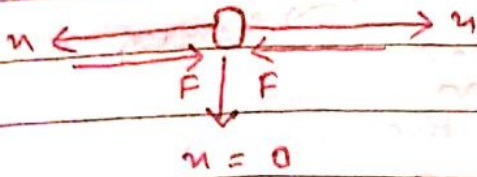
• All oscillatory motion are periodic (99%).



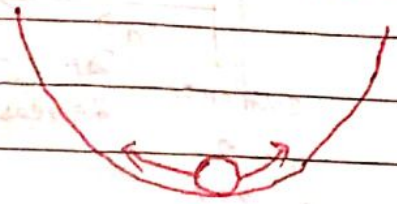
• Oscillatory \rightarrow equation : $F = -kx^n$

$n = 1, 3, 5, 7$

\downarrow
measured from mean position



Mean position (Stable Equilibrium position)



Force
 \downarrow
Mean position

"Yes. All oscillatory motions are periodic ~~to~~ except

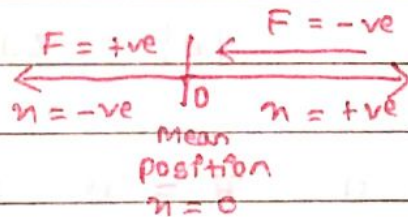
पे माला
-माला ए।

in which Energy is lost."

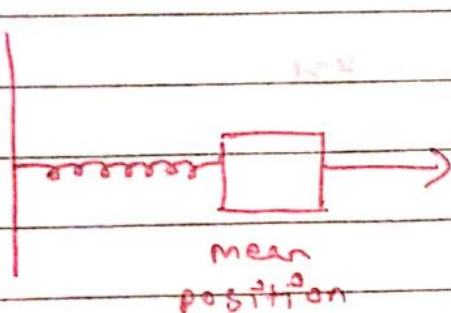
• SHM \rightarrow Special case / simplest case of Oscillation

St. line

$F = -kx$



$x \rightarrow$ measured from mean position



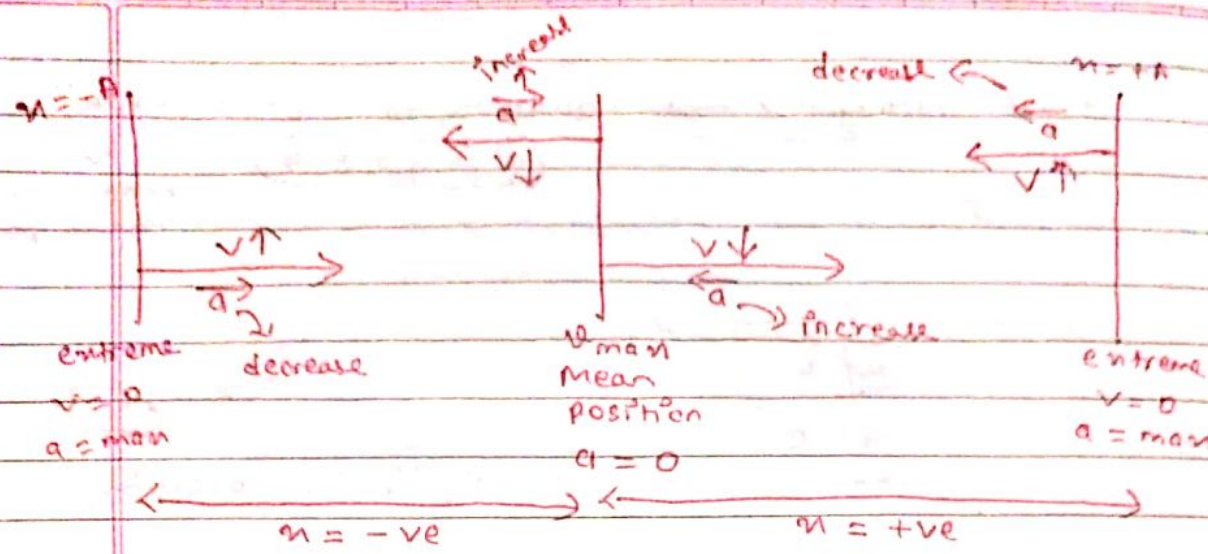
$\rightarrow a_{max} = \omega A$

extreme $x = \pm A$

$\rightarrow v = 0$ extreme

$v = \pm \omega \sqrt{A^2 - x^2}$

$\rightarrow v_{max} = \pm A\omega$
mean



$$F = -kx$$

$$a = \frac{F}{m} = -\frac{kx}{m}$$

$$\omega = \sqrt{\frac{k}{m}}$$

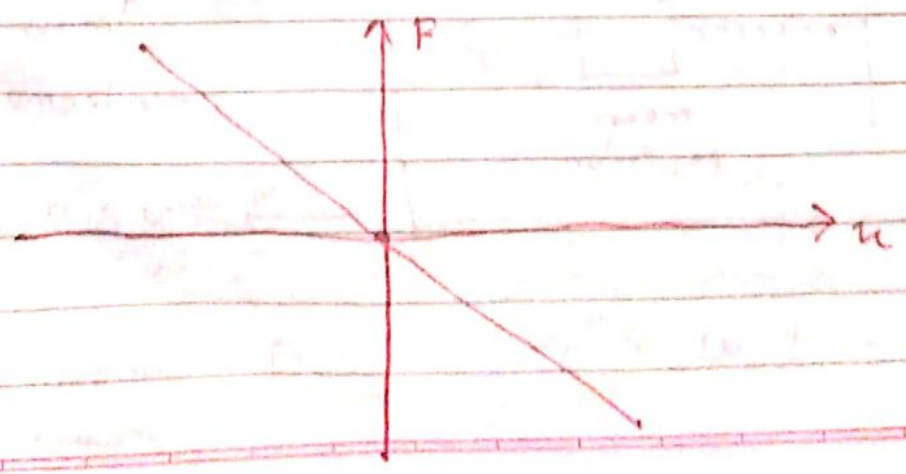
$$a = -\omega^2 x$$

$a \rightarrow \text{max}$, when $x = +A$ or $x = -A$
extreme position

$$|a| = \omega^2 A$$

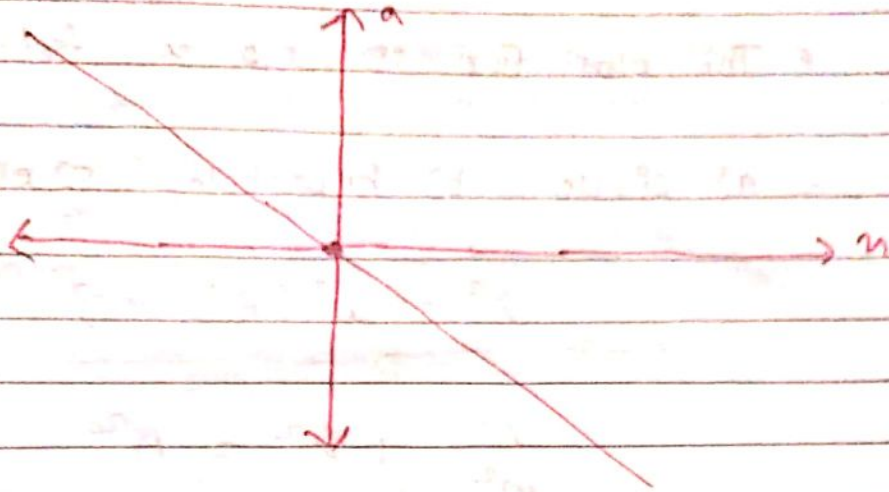
$a \rightarrow 0$, $x = 0$ (mean position)

• F vs x $F = -kx$



• a v/s x

$$a = -\omega^2 x$$



• v v/s x

$$a = -\omega^2 x$$

$$\frac{dv}{dt} \times \frac{dx}{dx} = -\omega^2 x$$

$$v \frac{dv}{dx} = -\omega^2 x$$

$$\int_0^v v \, dv = - \int_{x=A}^x \omega^2 x \, dx$$

$$\left[\frac{v^2}{2} \right]_0^v = -\omega^2 \left[\frac{x^2}{2} \right]_A^x$$

$$\frac{v^2}{2} = -\frac{\omega^2}{2} (x^2 - A^2)$$

$$v^2 = \omega^2 (A^2 - x^2)$$



• The plot for v vs x is: (in SHM)

a) circle b) hyperbola c) ellipse d) parabola

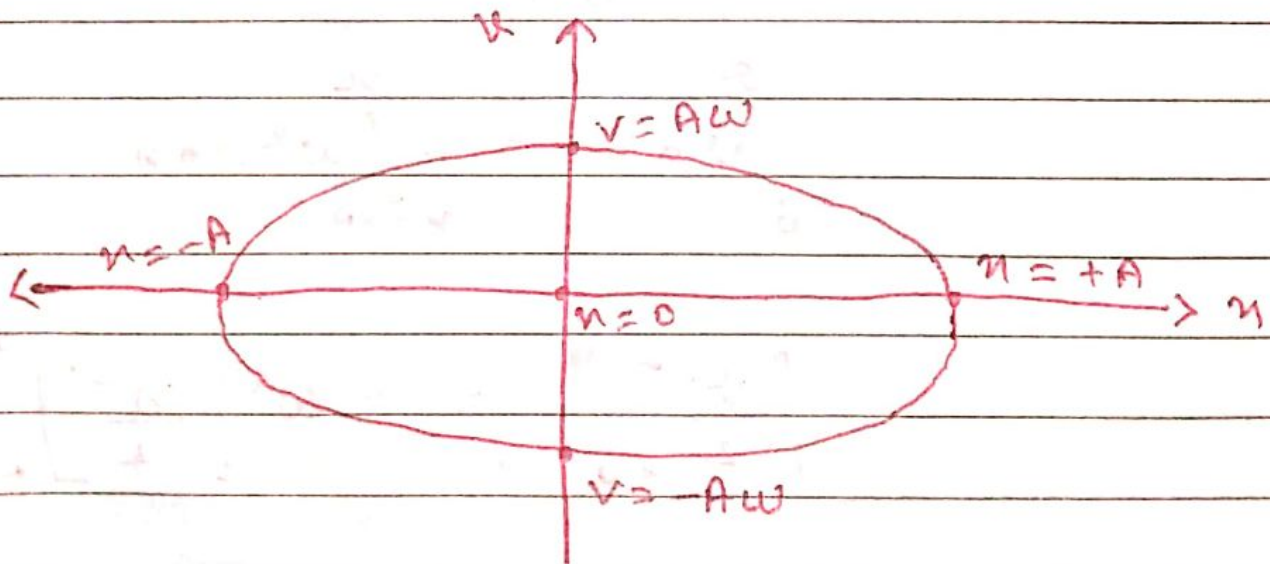
$$v^2 = \omega^2 (A^2 - x^2)$$

$$\frac{v^2}{\omega^2} + x^2 = A^2$$

$$\frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$

Ellipse $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right)$

$$\frac{x^2}{A^2} + \frac{v^2}{(\omega A)^2} = 1$$



$$V_{\max} = \pm A\omega$$

Mean position

$$v = \pm \omega \sqrt{A^2 - x^2}$$

$$\frac{dx}{dt} = \pm \omega \sqrt{A^2 - x^2}$$

$$\int_0^x \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \pm \omega dt$$

at $t=0$ initially
particle is at
mean position
 $x=0$

$$\text{Let } x = A \sin \theta$$

$$\sqrt{A^2 - x^2} = \sqrt{A^2 (1 - \sin^2 \theta)} = A \cos \theta$$

$$dx = A \cos \theta d\theta$$

$$\int_0^x \frac{A \cos \theta d\theta}{A \cos \theta} = \int_0^t \pm \omega dt$$

$$\left[\sin^{-1} \left(\frac{x}{A} \right) \right]_0^x = \pm \omega (t)_0^t$$

$$\sin^{-1} \frac{x}{A} = \pm \omega t$$

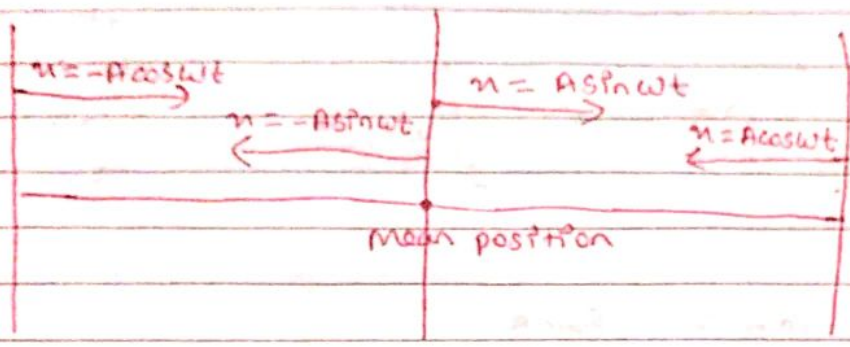
$$\frac{x}{A} = \sin(\pm \omega t)$$

$$\boxed{x = A \sin(\pm \omega t)} \quad \text{or}$$



$$x = A \sin \omega t \quad \text{or} \quad x = A \cos \omega t$$

$$x = -A \sin \omega t$$



• At $t = 0$ particle is at $x = +A$ (extreme)

$$\int_{+A}^x \frac{dx}{\sqrt{A^2 - x^2}} = \int_0^t \pm \omega dt$$

$$\left[\sin^{-1} \left(\frac{x}{A} \right) \right]_A^x = \pm \omega(t)_0^t$$

$$\sin^{-1} \left(\frac{x}{A} \right) - \sin^{-1} \left(\frac{A}{A} \right) = \pm \omega t$$

$$\sin^{-1} \left(\frac{x}{A} \right) - \frac{\pi}{2} = \pm \omega t$$

$$\sin^{-1} \left(\frac{x}{A} \right) = \frac{\pi}{2} \pm \omega t$$

$$\sin^{-1}(-1) = \sin \frac{3\pi}{2}$$

$$x = A \sin \left(\frac{\pi}{2} \pm \omega t \right)$$

$$x = A \cos \omega t$$

• At $t = 0$ particle is at $x = -A$

$$\left[\sin^{-1} \left(\frac{x}{A} \right) \right]_{-A}^A = \pm \omega t$$

$$\sin^{-1} \left(\frac{x}{A} \right) - \sin^{-1} \left(\frac{-A}{A} \right) = \pm \omega t$$

$$\sin^{-1} \left(\frac{x}{A} \right) - \frac{3\pi}{2} = \pm \omega t$$

$$\sin^{-1} \left(\frac{x}{A} \right) = \frac{3\pi}{2} \pm \omega t$$

$$x = A \sin \left(\frac{3\pi}{2} \pm \omega t \right)$$

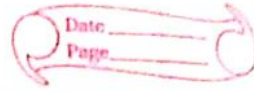
$$x = -A \cos \omega t$$

General eqⁿ :-

$$x = A \sin(\omega t + \phi)$$
$$x = A \cos(\omega t + \phi)$$

$$x = A \sin(\omega t + \phi)$$

$$x = A \cos(\omega t + \phi)$$



$\phi \rightarrow$ initial phase, $(\omega t + \phi) \rightarrow$ phase

$$x = A \sin(\omega t + \phi)$$

$$\frac{dx}{dt} = A \omega \cos(\omega t + \phi)$$

$$v = A \omega \cos(\omega t + \phi)$$

$$a = \frac{dv}{dt} = A \omega^2 (-\sin(\omega t + \phi))$$

$$a = -A \omega^2 \sin(\omega t + \phi)$$

$$a = -\omega^2 x \quad \text{differential equation of SHM}$$

$$\frac{dv}{dt} = -\omega^2 x$$

$$\frac{d\left(\frac{dx}{dt}\right)}{dt} = -\omega^2 x$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$



$$v_{\max} = \pm A\omega$$

$$a_{\max} = \pm A\omega^2$$

• Time period of 'SHM'

$$x_1 = x_2$$

$$A \sin(\omega t + \phi) = A \sin(\omega(t+T) + \phi)$$

$$\sin \theta = \sin \alpha$$

$$\theta = \alpha, \quad \theta = 2\pi + \alpha$$

$$\theta = \pi - \alpha$$

$$v = A\omega \cos(\omega t + \phi)$$

$$v_1 = v_2$$

$$A\omega \cos(\omega t + \phi) = A\omega \cos(\omega(t+T) + \phi)$$

$$\cos \theta = \cos \alpha,$$

$$\theta = \alpha, \quad \theta = 2\pi + \alpha$$

$$\theta = 2\pi - \alpha$$

$$\cancel{\omega t} + \phi + 2\pi = \cancel{\omega t} + \omega T + \phi$$

$$\omega T = 2\pi$$

$$T = \frac{2\pi}{\omega}$$

Q) Write the eqⁿ for SHM ($x-t$) if initially particle is at origin & moving in -ve direction w.r.t. mean position.

(Mean position = origin) ($A = 2\text{cm}$, $T = 4\text{s}$)

$$\Rightarrow T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$x = -2\sin\left(\frac{\pi}{2}t\right)$$

$$\boxed{x = -2\sin\left(\frac{\pi}{2}t\right)}$$

OR

At $t = 0$
 $x = 0$

$$x = A\sin(\omega t + \phi) \qquad v = A\omega\cos(\omega t + \phi)$$

$$0 = A\sin(0 + \phi) \qquad v < 0$$

$$t = 0$$

$$\sin\phi = 0 \qquad A\omega\cos(0 + \phi) < 0$$

$$\phi = 0 \text{ or } \pi \qquad \cos\phi < 0$$

$$\cos 0 = 1 \qquad \cos 180^\circ = -1$$

$$\boxed{\phi = \pi}$$

$$x = 2\sin(\omega t + \pi)$$

$$\boxed{x = -2\sin\left(\frac{\pi}{2}t\right)}$$

$$0 \leq \phi \leq 2\pi$$

Q) Write the eqⁿ for SHM (x-t), (v-t)
 & find 'v' at t = 2sec, 3sec if
 initially particle is at right extreme of
 origin & moving in -ve x-direction.
 (A = 2cm, T = 8sec)

⇒

$$x = A \sin(\omega t + \phi) \quad v = A \omega \cos(\omega t + \phi)$$

At t = 0
x = A

$$A = A \sin(0 + \phi) \quad v = A \omega \cos\left(\frac{\pi}{2} + \omega t\right)$$

$$\sin \phi = 1$$

$$\phi = \frac{\pi}{2}$$

$$v = -A \omega \sin(\omega t)$$

$$= -\frac{2 \times \pi}{4 \times 2} \sin\left(\frac{\pi}{4} t\right)$$

$$x = A \sin\left(\frac{\pi}{2} + \omega t\right)$$

$$= -\frac{\pi}{2} \sin\left(\frac{\pi}{4} t\right)$$

$$= A \cos \omega t$$

$$T = \frac{2\pi}{\omega}$$

At t = 2sec,

$$v = -\frac{\pi}{2} \sin\left(\frac{\pi}{2}\right)$$

$$4 \times 8 = \frac{2\pi}{\omega}$$

$$v = -\frac{\pi}{2}$$

$$\omega = \frac{\pi}{4}$$

At t = 3sec,

$$v = -\frac{\pi}{2} \sin\left(\frac{3\pi}{4}\right)$$

$$x = 2 \cos\left(\frac{\pi}{4} t\right)$$

$$v = -\frac{\pi}{2\sqrt{2}}$$

$$x = 2 \cos\left(\frac{\pi}{4} t\right)$$

At t = 4sec, $v = 0$

$t_1 < t_2$

Date _____

Page _____

Q.3) Find $v-t$ for SHM, if initially particle is at $+\frac{A}{\sqrt{2}}$ of origin & moving initially in +ve direction.

$$\Rightarrow x = A \sin(\omega t + \phi) \quad v = A\omega \cos(\omega t + \phi)$$

At $t=0$

$$x = \frac{A}{\sqrt{2}}$$

$$\frac{A}{\sqrt{2}} = A \sin \phi$$

$$v = A\omega \cos(\omega t + \phi) > 0$$

$$\cos(\phi) > 0$$

$$\sin \phi = \frac{1}{\sqrt{2}}$$

$$\phi = \frac{\pi}{4}$$

$$\phi = \frac{\pi}{4}$$

$$v = A\omega \cos\left(\omega t + \frac{\pi}{4}\right)$$

IIT 2001

If initially particle is at origin (SHM) & it covers $0 \rightarrow \frac{A}{2}$ in t_1 time & $\frac{A}{2} \rightarrow A$ in t_2 time.

a) $t_2 = t_1$ b) $t_2 = t_1/2$

c) $t_2 = 2t_1$ d) $t_2 = t_1 + 2$

$$\Rightarrow x = A \sin \omega t$$

$$\frac{A}{2} = A \sin \omega t_1$$

$$\sin \omega t_1 = \frac{1}{2} \quad \text{---} \quad x' = A$$

$$t = (t_1 + t_2)$$

$$\omega t_1 = \frac{\pi}{6}$$

$$A = A \sin \omega (t_1 + t_2)$$

$$t_1 = \frac{\pi}{6\omega}$$

$$\omega (t_1 + t_2) = \frac{\pi}{2}$$

$$t_2 = \frac{\pi}{3\omega}$$

$$\omega t_2 = \frac{\pi}{2} - \frac{\pi}{6}$$

$$\omega t_2 = \frac{\pi}{3}$$

JEE Mains 2014

SHM \rightarrow A particle starts from rest & covers 'a' distance in 'T' sec & another '2a' distance in next 'T' sec in the same direction. Which of the following is correct?

- a) $A = 3a$ b) $A = 4a$
 c) $T = 6T$ d) $T = 8T$

\Rightarrow

$$x = A \cos \omega t$$

At $t = 2T$
 $x = 3a$

At $t = T$
 $x = a$

$$3a = A \cos 2\omega T$$

$$a = A \cos \omega T$$

$$3 \times A \cos \omega T = A \cos 2\omega T$$

$$3 \cos \omega T = \cos 2\omega T$$

$$x = A \cos(\omega t + \phi)$$

$E = 0$
 $v = 0$
extreme

$$\Rightarrow \text{At } t = \tau,$$
$$x = A - a$$

$$A - a = A \cos \omega t$$

$$At, t = 2T$$

$$x = A - 3a$$

$$\cos \omega t = \frac{A-a}{A}$$

$$A - 3a = A \cos \omega 2T$$

$$x = A (2 \cos^2 \omega t - 1)$$

$$A - 3a = A \left(2 \left(\frac{A-a}{A} \right)^2 - 1 \right)$$

$$A - 3a = A \left[\frac{(A^2 + a^2 - 2Aa)2}{A^2} - 1 \right]$$

$$A - 3a = A \left[\frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2} \right]$$

$$A - 3a = \frac{A^2 + 2a^2 - 4Aa}{A}$$

$$Aa = 2a^2$$

$$A = 2a$$

$$A - a = A \cos \omega t$$

$$x = 2a \cos \omega t$$

$$\cos \omega t = \frac{1}{2}$$

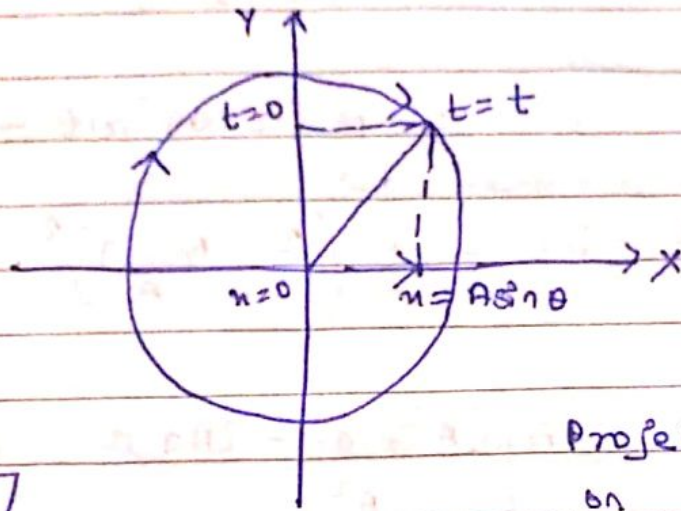
$$\omega t = \frac{\pi}{3}$$

$$\omega t = \frac{\pi}{3}$$

$$\frac{2\pi}{T} \times t = \frac{\pi}{3}$$

$$T = 6\pi \cdot \omega$$

• SHM & Circular Motion :



Projection is
doing to &
from motion

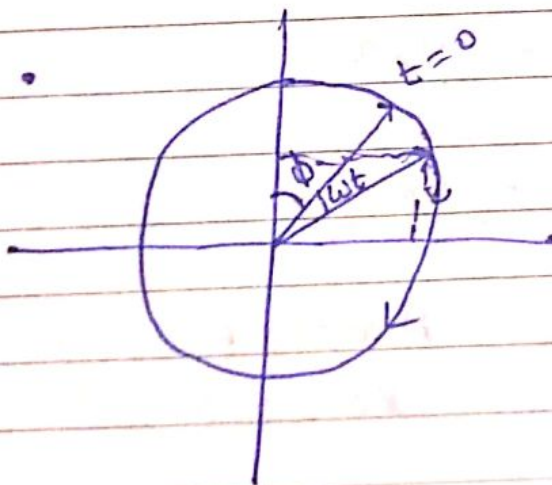
Projection of particle
on diameter

$$\theta = \omega t$$

$$x = A \sin \theta$$

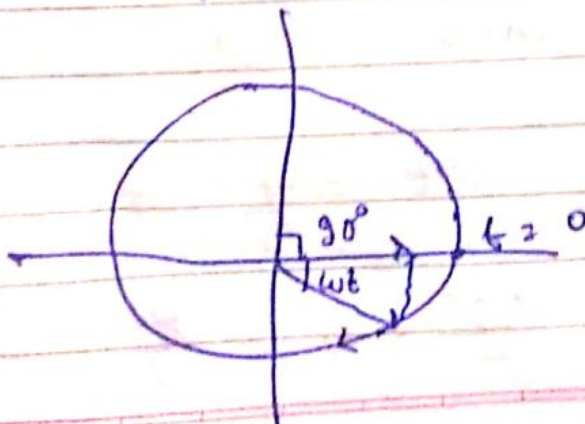
$$x = A \sin \omega t$$

Yes SHM

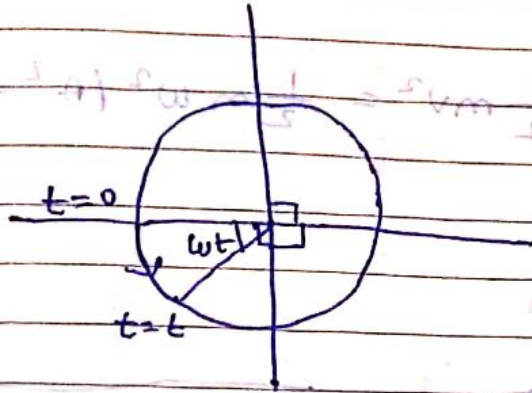
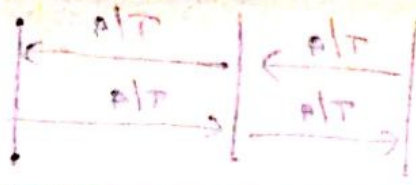


$$x = A \sin (\omega t + \phi)$$

Equation for
SHM



$$x = A \cos \omega t$$



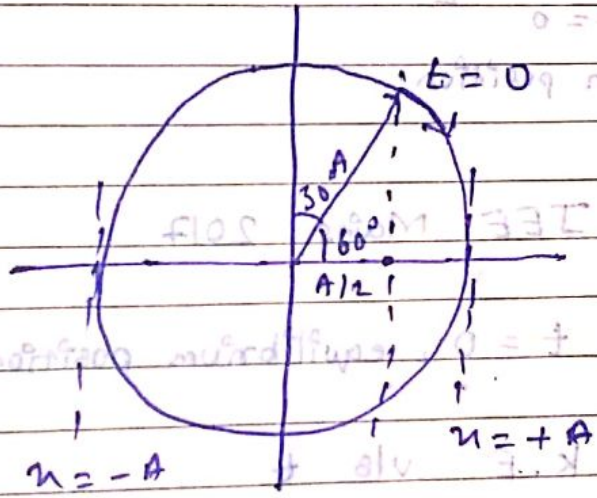
$$x = A \sin(\omega t + \phi)$$

$$x = A \sin(270 - \omega t)$$

$$x = -A \cos \omega t$$

Q) SHM $\therefore t=0$ initially $x = +\frac{A}{2}$ & initially

it is moving in the x direction. find equation for



$$x = A \sin(\omega t + \phi)$$

$$x = A \sin(\omega t + \frac{\pi}{6})$$

SHM

$$F = -kx$$

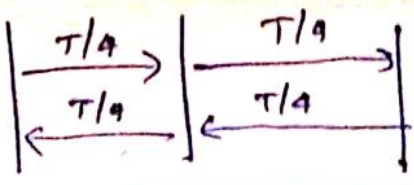
↳ conservative

Kinetic Energy (Variable)

Potential Energy (Variable)

Total Mechanical Energy

↓
Constant



$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 (A^2 - x^2)$

$(\omega - \text{of } \dots) \Rightarrow A \omega^2 = \frac{k}{m}$

$\therefore K.E = \frac{1}{2} k (A^2 - x^2)$

Max $= \frac{1}{2} k A^2$
 $x = 0$

Min $= 0$

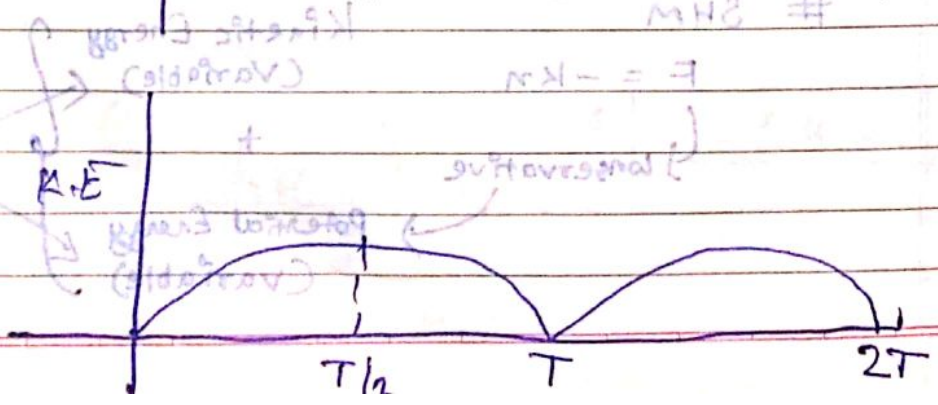
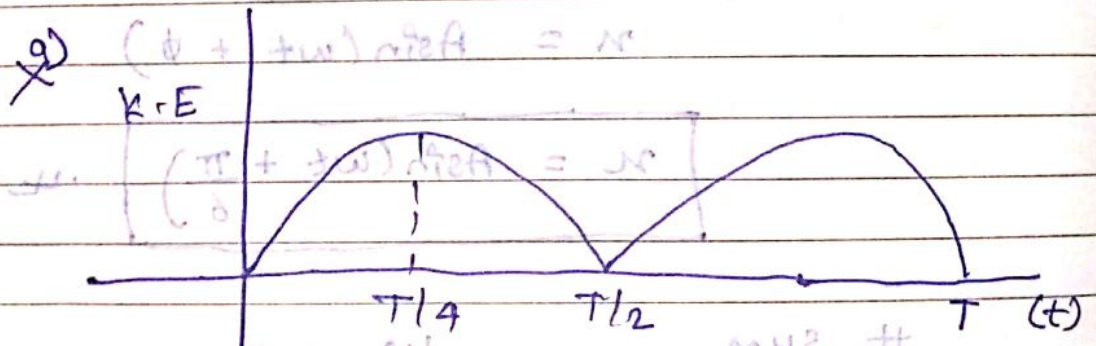
Mean position

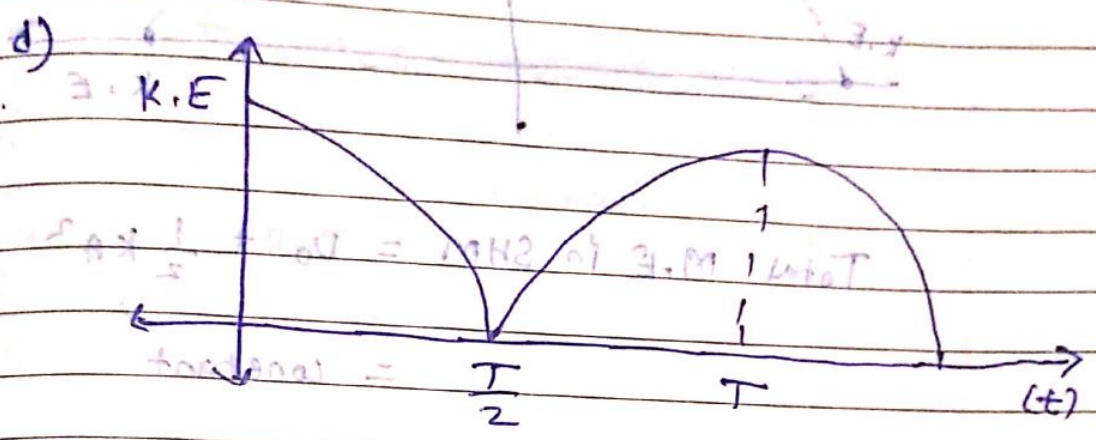
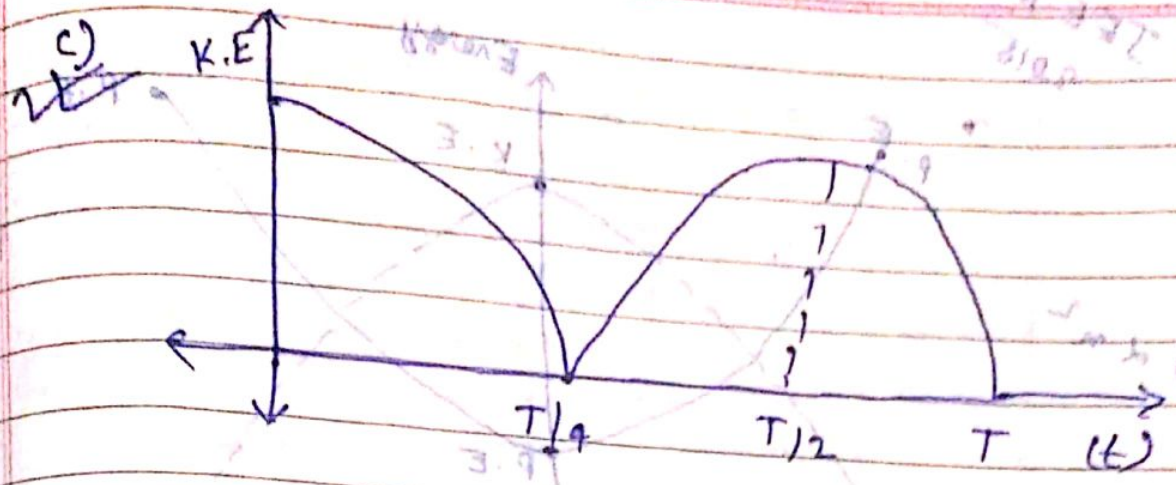
extreme position
 $x = \pm A$

JEE Mains 2017

$t = 0$, equilibrium position (Mean position)

K.E vs t





$\Delta U = -W_{\text{conservative force}}$
 \downarrow
 $F = -kx$

conservative force
 $\int dU = - \int dW_c$
 $\int_{U_0}^U dU = - \int_{x=0}^{x=u} kx dx \cos 180^\circ$
 $U - U_0 = \frac{1}{2} kx^2$

$U - U_0 = \frac{1}{2} kx^2$

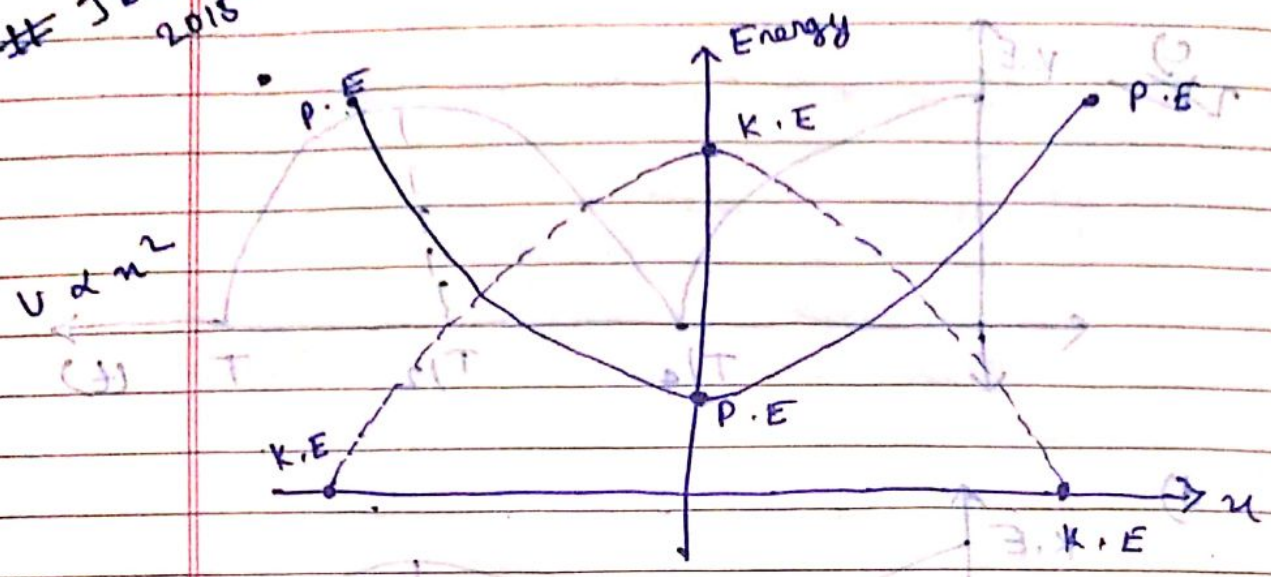
$U = U_0 + \frac{1}{2} kx^2$

$x=0$ mean position $U_{\min} = U_0$
 $x = \pm A$ extreme $U_{\max} = U_0 + \frac{1}{2} kA^2$

$F = -kx$
 $a = -\frac{kx}{m}$
 $-\frac{k}{m} = \omega^2$

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JEE Mains 2015



Total M.E in SHM = $U_0 + \frac{1}{2}kA^2$

= constant

Q) Assuming $U = 0$ at mean position
 Find Total M.E of particle undergoing SHM of mass 2 kg,

$x = 2 \sin\left(\frac{\pi}{4}t + \frac{\pi}{4}\right)$

$A = 2$
 $\omega = \frac{\pi}{4}$

$m = 2 \text{ kg}$

$k = \omega^2 m$

$= \frac{\pi^2}{16} \times 2$

$= \frac{\pi^2}{8}$

$A \pm = m, \text{ M.M}$

$$T.E = \frac{1}{2} k A^2$$

$$= \frac{1}{2} \times \frac{\pi^2}{8} \times (4)^2$$

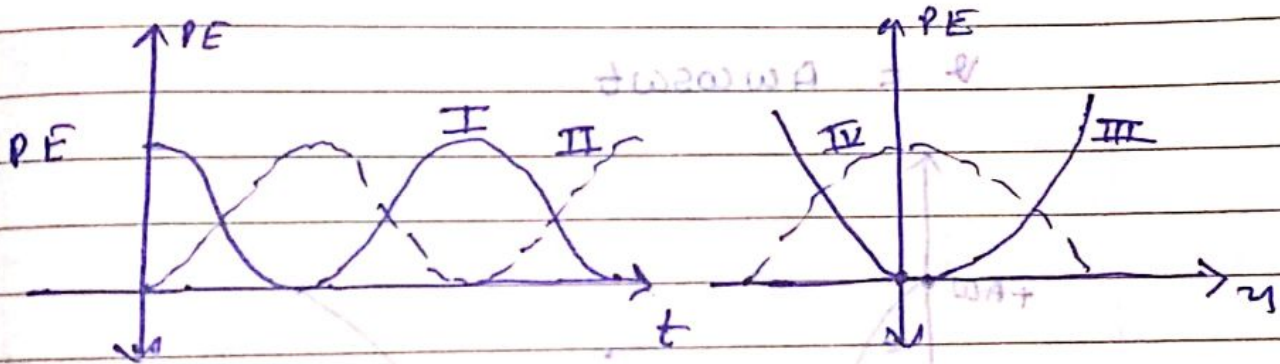
$$= \frac{\pi^2}{4} \text{ J}$$

IIT 2003:

SHM, $x = A \cos \omega t$

P.E. v/s t

P.E. v/s x



a) I & IV b) II & III c) I & III d) II & IV

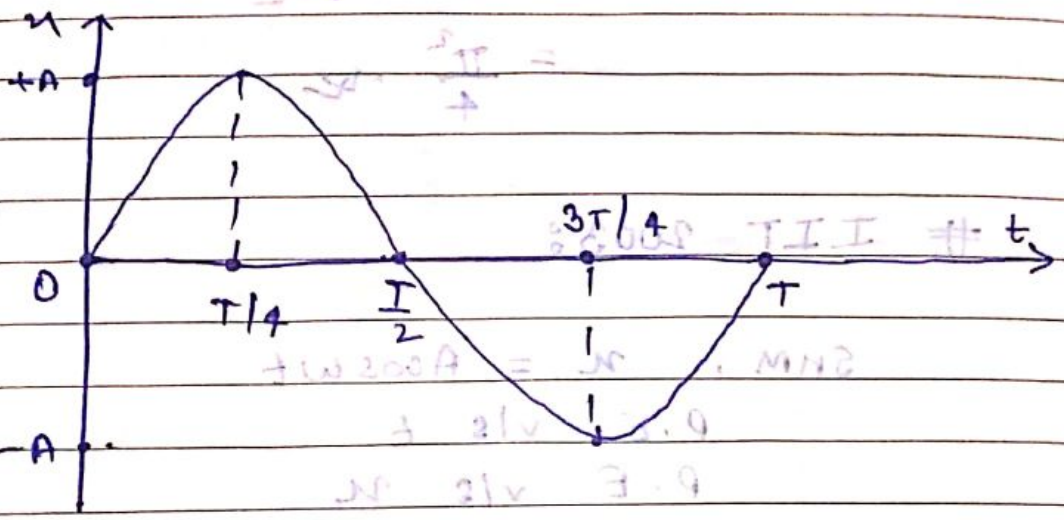
$$P.E = \frac{1}{2} k x^2 = \frac{1}{2} k A^2 \sin^2 (\omega t + \phi)$$

$$K.E = \frac{1}{2} m v^2 = \frac{1}{2} m A^2 \omega^2 \cos^2 (\omega t + \phi)$$

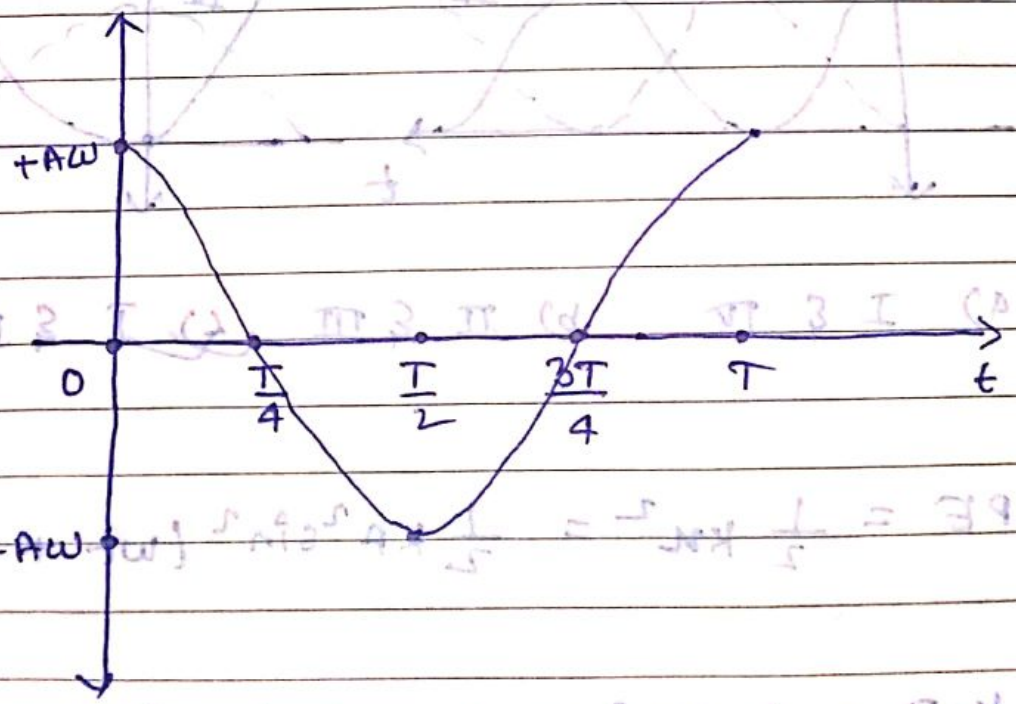
$$= \frac{1}{2} k A^2 \cos^2 (\omega t + \phi)$$

$$T.E = \frac{1}{2} k A^2$$

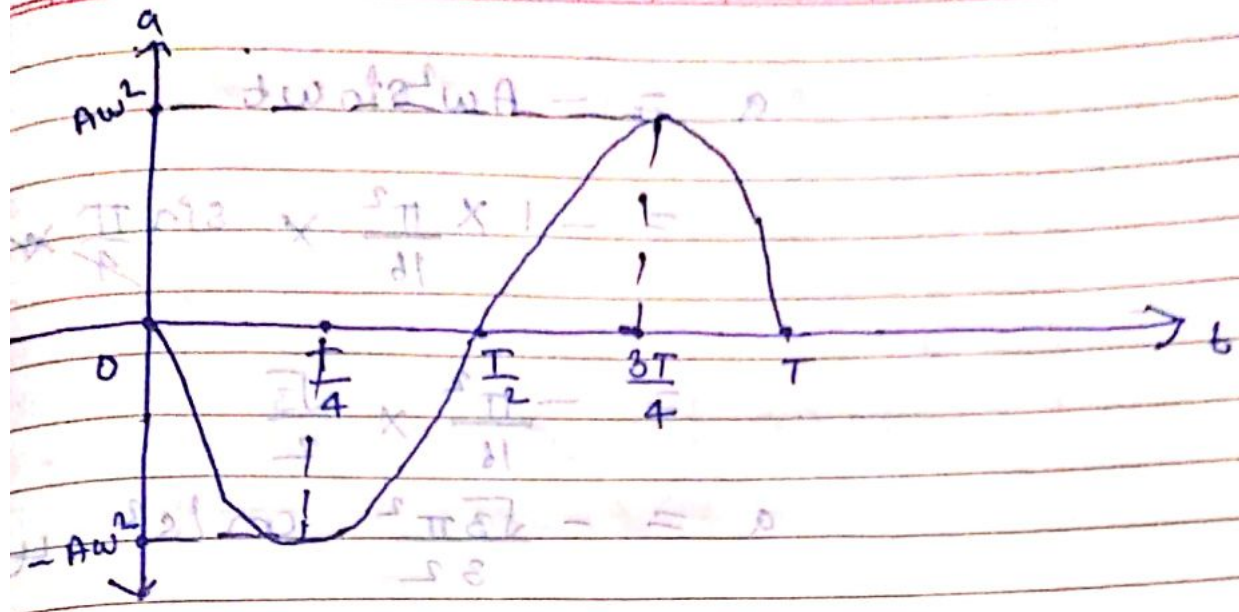
• $x = A \sin \omega t$ $t = 0$
 $x = A \sin \frac{2\pi}{T} \times t$ $x = 0$
+ve direction motion



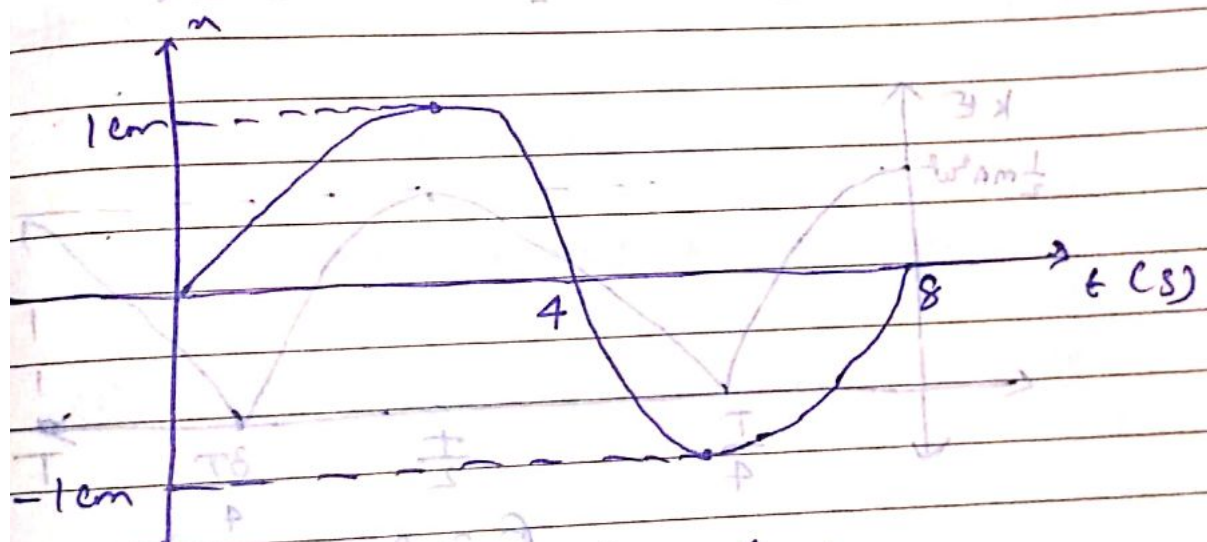
$v = -A\omega \cos \omega t$



$a = -A\omega^2 \sin \omega t = -A\omega^2 \sin \frac{2\pi}{T} t$
 $= -\omega^2 x$



IIT 2009



a) when $t = \frac{4}{3} s$

a) $\frac{\pi^2}{32} \text{ cm/s}^2$ b) $-\frac{\pi^2}{32} \text{ cm/s}^2$ c) $\frac{\sqrt{3}\pi^2}{32} \text{ cm/s}^2$

d) $\frac{\sqrt{3}\pi^2}{32} \text{ cm/s}^2$

$\Rightarrow T = 8 \text{ sec}$ $A = 1 \text{ cm}$ $x = A \sin \omega t$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$$

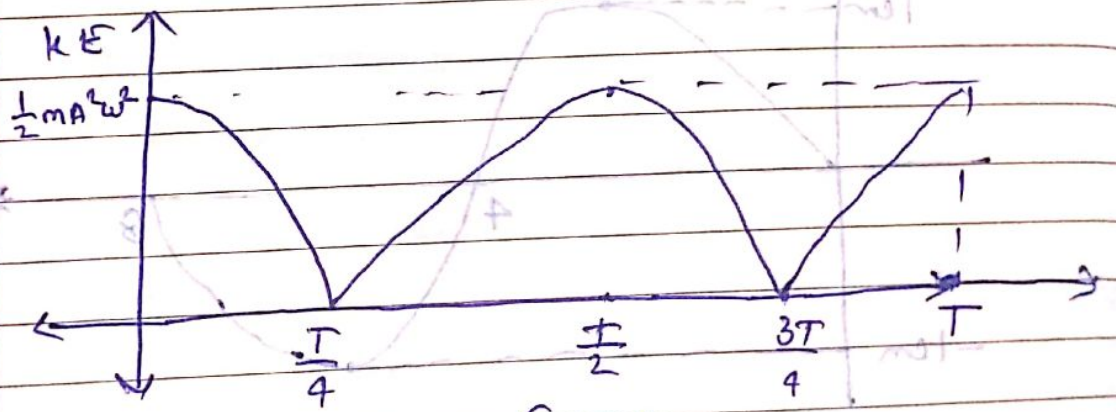
$$a = -A\omega^2 \sin \omega t$$

$$= -1 \times \frac{\pi^2}{16} \times \sin \frac{\pi}{4} \times \frac{1}{3}$$

$$= -\frac{\pi^2}{16} \times \frac{\sqrt{3}}{2}$$

$$a = -\frac{\sqrt{3}\pi^2}{32} \text{ cm/s}^2$$

$$\bullet \text{ K.E} = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2 \cos^2 \omega t$$



$$f = 2$$

In SHM, $x = A \sin \omega t$ (if $\omega = 1$)

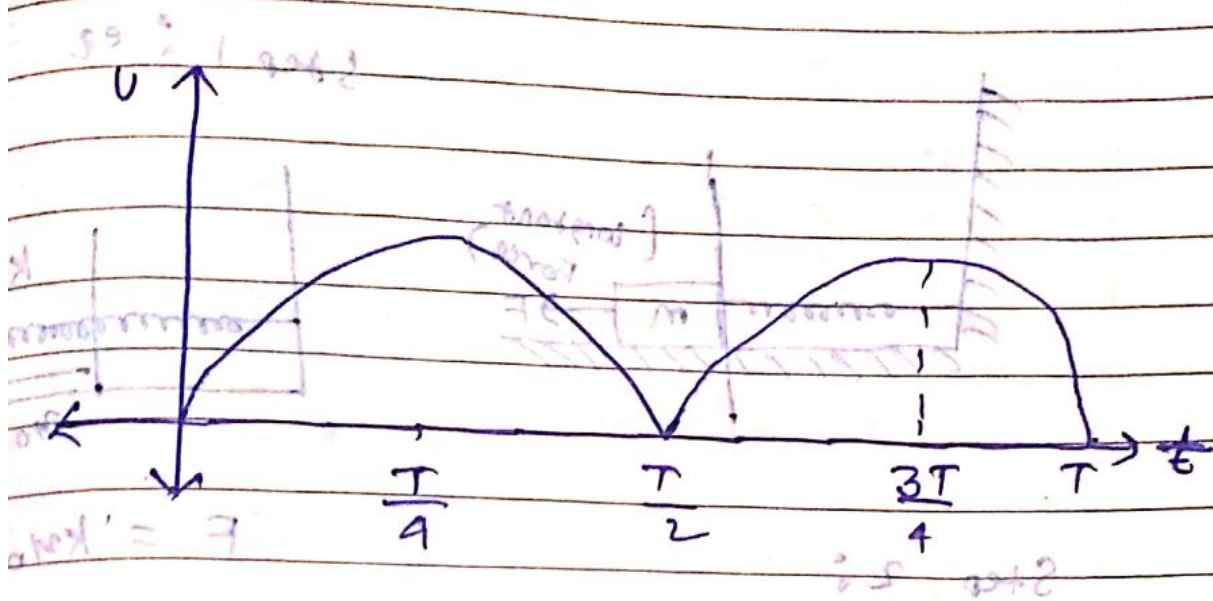
$v \rightarrow$ SHM $A\omega \cos \omega t$ (ω)

$a \rightarrow$ SHM $-A\omega^2 \sin \omega t$ (ω)

K.E \rightarrow SHM Periodic (2ω)

P.E \rightarrow SHM Periodic (2ω)

$$U = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$$



Steps to Find Time Period of any S.H.M :-

(1) Find Equilibrium position $F_{net} = 0$

Write balanced force equation.

(2) Displace the object slightly (x) from mean position & find new F_{net} & mathematically

arrive at $F_{net} = -kx$

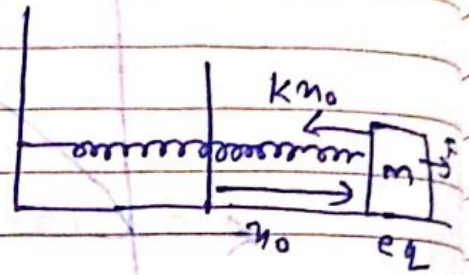
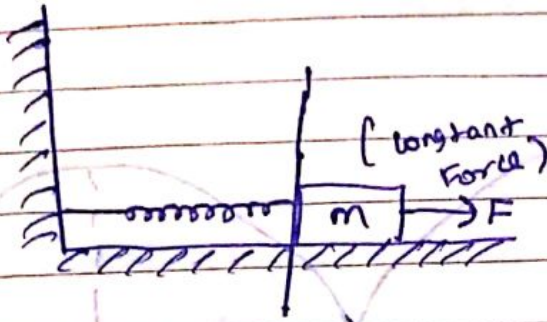
$$k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

SHM constant
Spring constant only
for spring block
system.

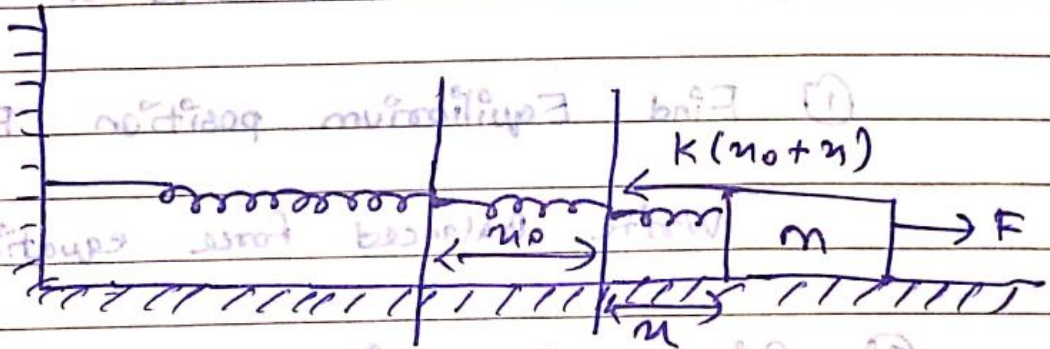
• Time Period of Spring Block System :

Step 1 : eq



$$F = kx_0$$

Step 2 :

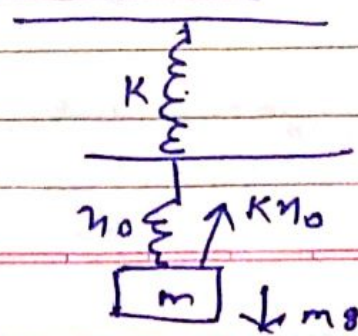
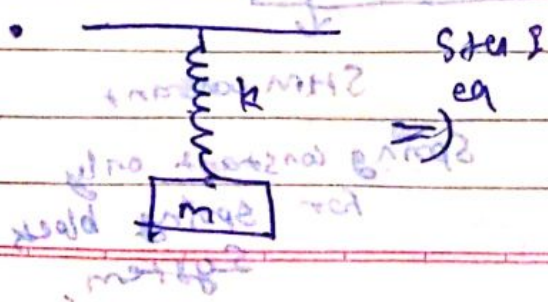


$$F_{net} = F - k(x_0 + x)$$

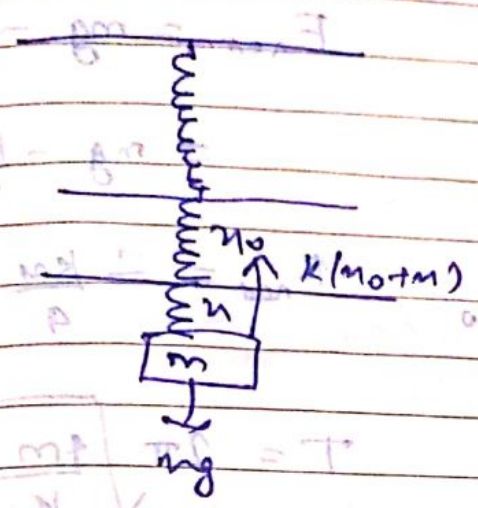
$$F_{net} = F - kx_0 - kx$$

$$F_{net} = -kx$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$



$mg = kn_0$



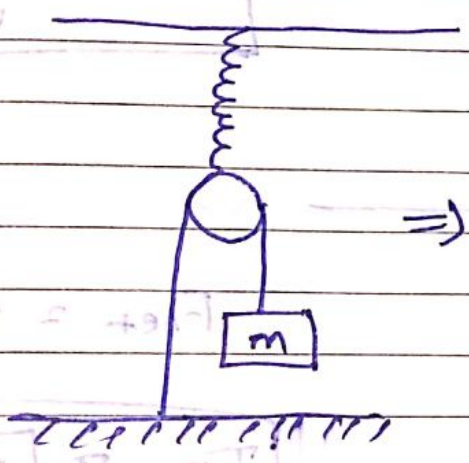
$F_{net} = mg - kn_0 - kx$

$= kn_0 - kn_0 - kx$

$F_{net} = -kx$

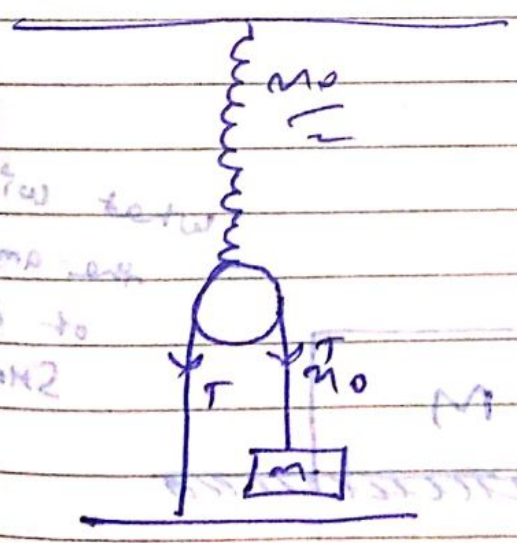
$T = 2\pi \sqrt{\frac{m}{k}}$

3)



~~$F_{net} = -\frac{2kx}{2}$~~

~~$T = 2\pi \sqrt{\frac{2m}{k}}$~~

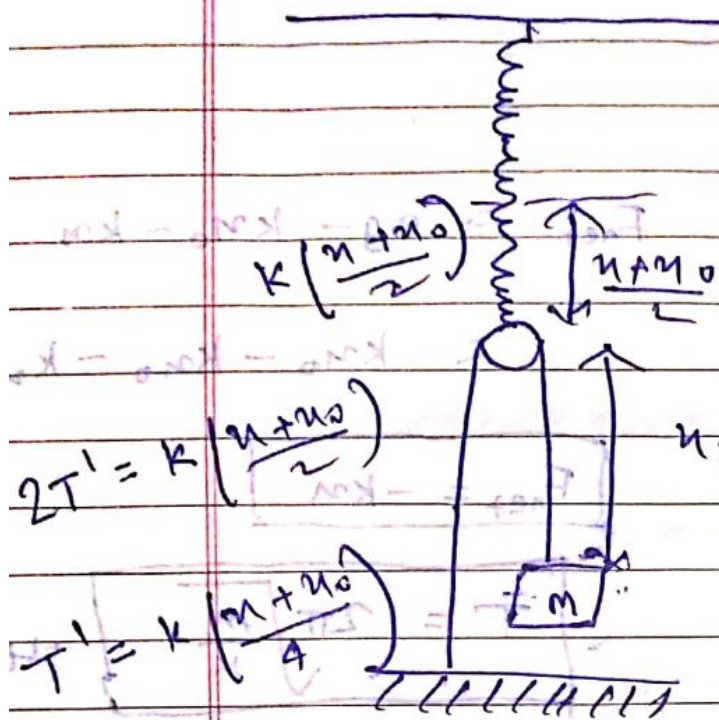


$2T = \frac{kx_0}{2}$

$T = \frac{kx_0}{4}$

$mg = T$

$mg = \frac{kx_0}{4}$



$$F_{net} = mg - T'$$

$$= mg - \frac{kx}{4} - \frac{kx_0}{4}$$

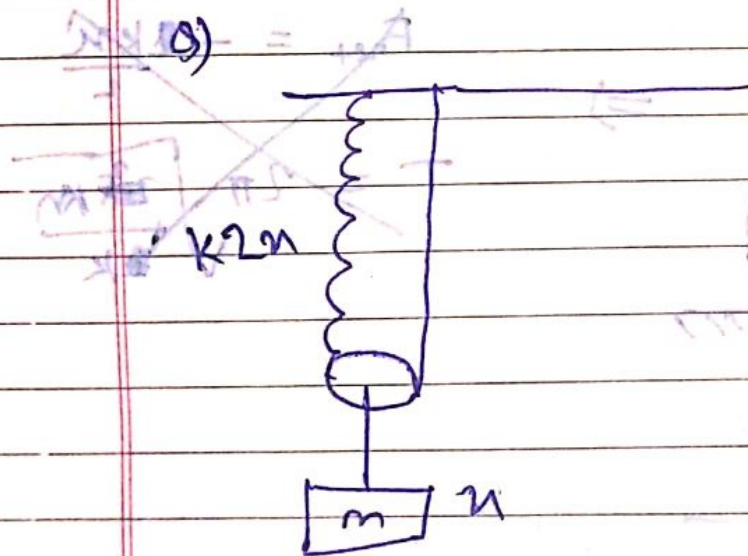
$$2T' = k\left(\frac{x + x_0}{2}\right)$$

$$T' = k\left(\frac{x + x_0}{4}\right)$$

$$F_{net} = -\frac{kx}{4}$$

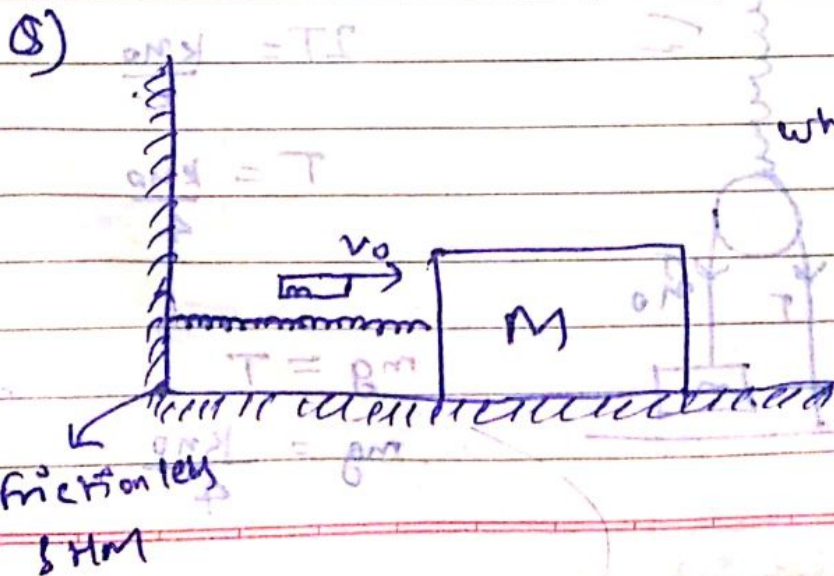
$$T = 2\pi \sqrt{\frac{4m}{k}}$$

$$T = 4\pi \sqrt{\frac{m}{k}}$$



$$F_{net} = 4kx$$

$$T = \pi \sqrt{\frac{m}{k}}$$



What will be the amplitude of that SHM??

$$mv_0 = (m + M) v'$$

$$v' = \frac{mv_0}{m + M}$$

~~$$\frac{1}{2} \frac{m^2 v_0^2}{m + M} = \frac{1}{2} k A^2$$

$$m^2 v_0^2 = (m + M) k A^2$$

$$v = \frac{mv_0}{\sqrt{(m + M)k}}$$~~

$$k = (M + m) \omega^2$$

$$\omega = \sqrt{\frac{k}{m + M}}$$

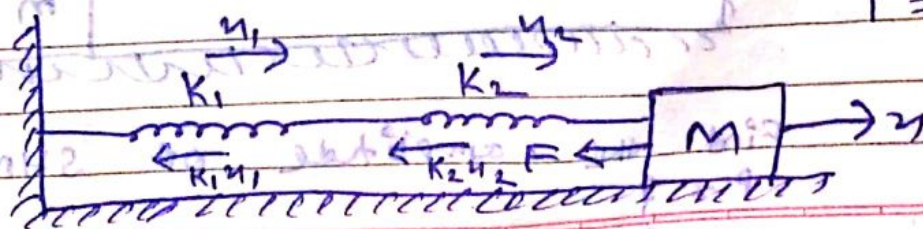
$$v_{\text{mean max}} = \omega A$$

$$\frac{mv_0}{m + M} = \omega A$$

$$\frac{mv_0}{m + M} = \sqrt{\frac{k}{m + M}} A$$

$$A = \frac{mv_0}{\sqrt{(m + M)k}}$$

• Series

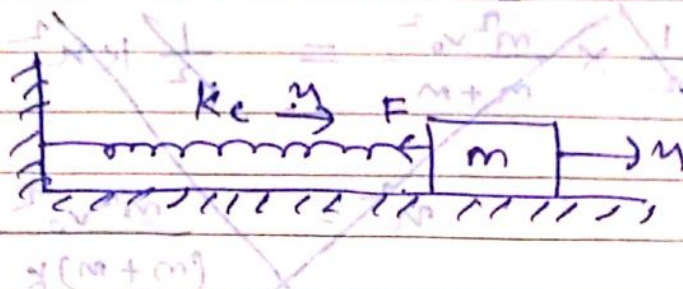


$$T = 2\pi \sqrt{\frac{m}{k_e}}$$

$K \uparrow \uparrow \Rightarrow$ Spring SHM

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$m K_e \omega^2 = \frac{K_1 K_2}{K_1 + K_2}$$



$$F = K_e x = K_1 x_1 = K_2 x_2$$

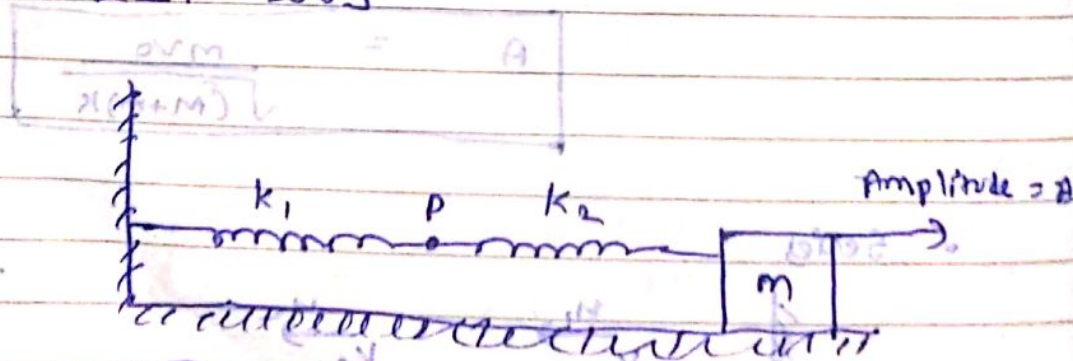
$$K_2 x_2 = F = K_1 x_1$$

$$x = x_1 + x_2$$

$$\frac{F}{K_e} = \frac{F}{K_1} + \frac{F}{K_2}$$

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

IIT 2009



Find the amplitude of SHM of P = ?

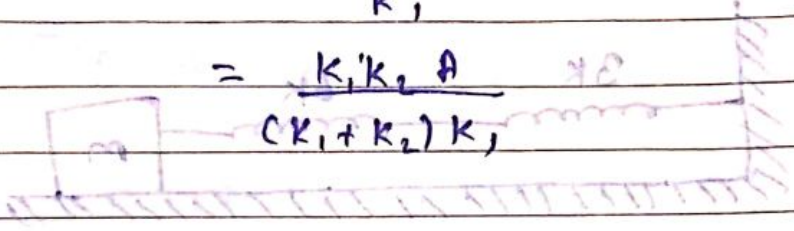
a) $\frac{k_1 A}{k_1 + k_2}$ b) $\frac{k_1 A}{k_1 + k_2}$ c) $\frac{k_1 A}{k_2}$ d) A

$m_1 x + m_2 x = m_3 x$

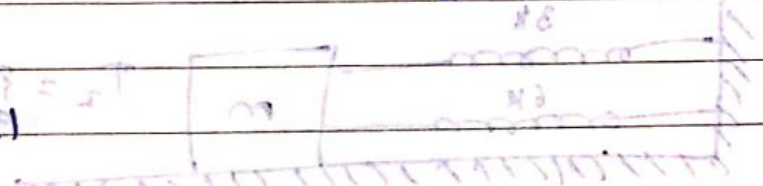
$\Rightarrow F = k_1 x_1 = k_2 x_2 = k_E A$

$x_1 = \frac{k_E A}{k_1}$ (1)

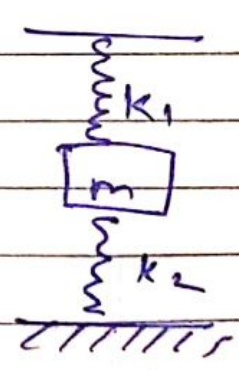
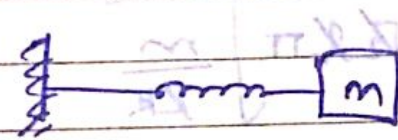
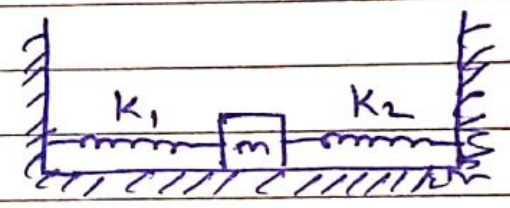
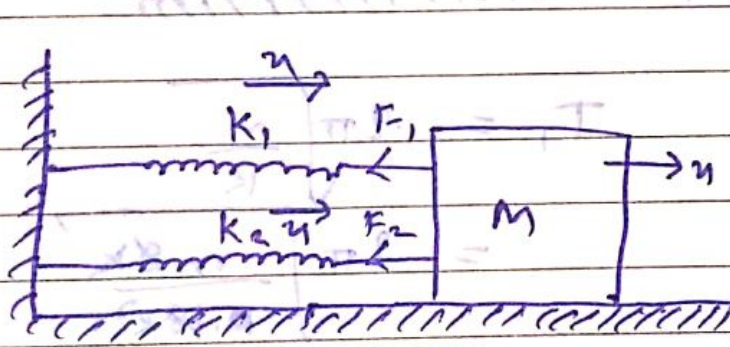
$= \frac{k_1 k_2 A}{(k_1 + k_2) k_1}$



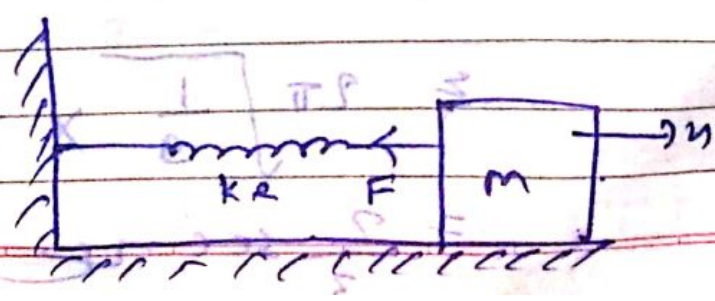
$x_1 = \frac{k_2 A}{k_1 + k_2}$ (2)



Parallel



$k_E = k_1 + k_2$

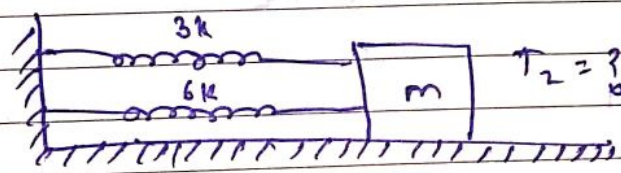
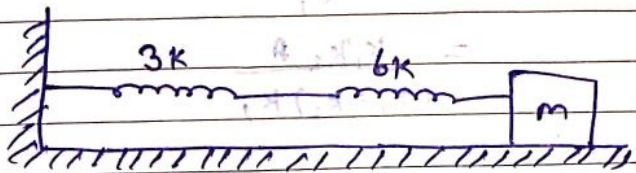


$$F = F_1 + F_2$$

$$k_e m = k_1 m + k_2 m$$

$$k_e = k_1 + k_2$$

8)



$$T_1 = 2\pi \sqrt{\frac{m}{k}}$$

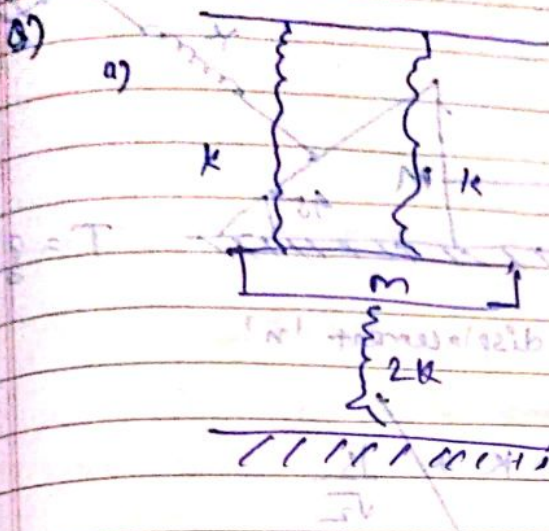
$$= 2\pi \sqrt{\frac{m \cdot 8k}{18k}}$$

$$\sqrt{2} = 2\pi \sqrt{\frac{m}{9k}}$$

$$T_2 = 2\pi \sqrt{\frac{m}{9k}}$$

$$= 2\pi \sqrt{\frac{1}{9}} \times \frac{1}{\pi}$$

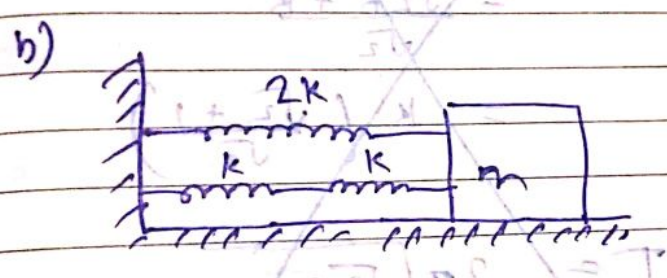
$$= \frac{2}{3} \text{ sec}$$



$$T = 2\pi \sqrt{\frac{m}{4k}}$$

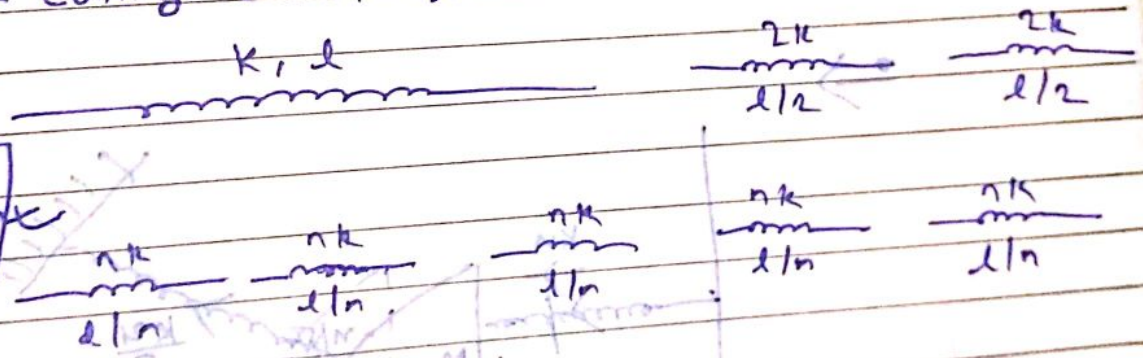
$$T_1 = \pi \sqrt{\frac{m}{k}}$$

Small Horizontal displacement x

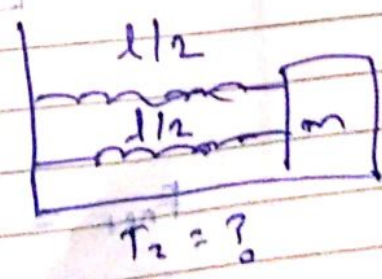
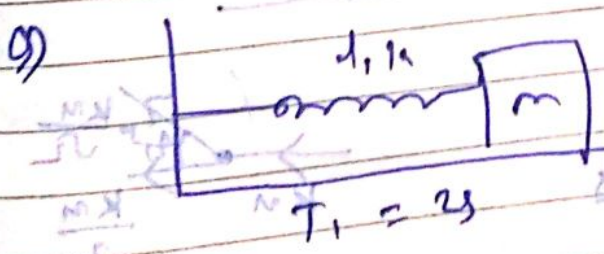


$$T_2 = 2\pi \sqrt{\frac{2m}{5k}}$$

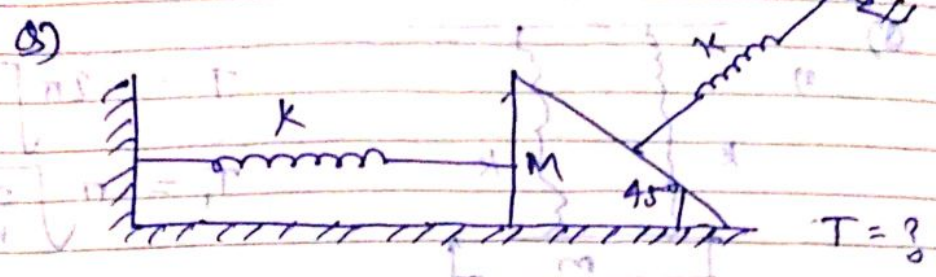
Cutting of Springs



$$k \propto \frac{1}{l}$$



$$T_2 = 2\pi \sqrt{\frac{2m}{5k}}$$



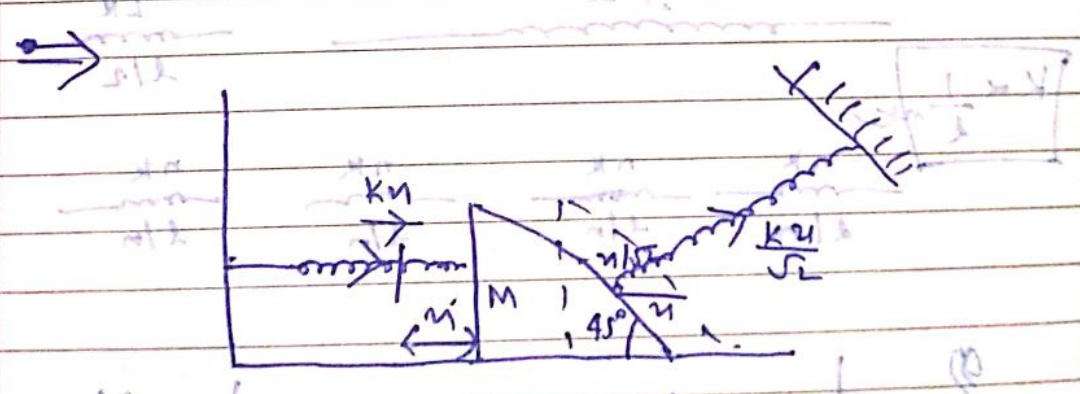
Small horizontal displacement 'x'

~~$$k_e = k + \frac{k}{\sqrt{2}}$$

$$= \frac{\sqrt{2}k + k}{\sqrt{2}}$$

$$= k \left(\frac{\sqrt{2} + 1}{\sqrt{2}} \right)$$

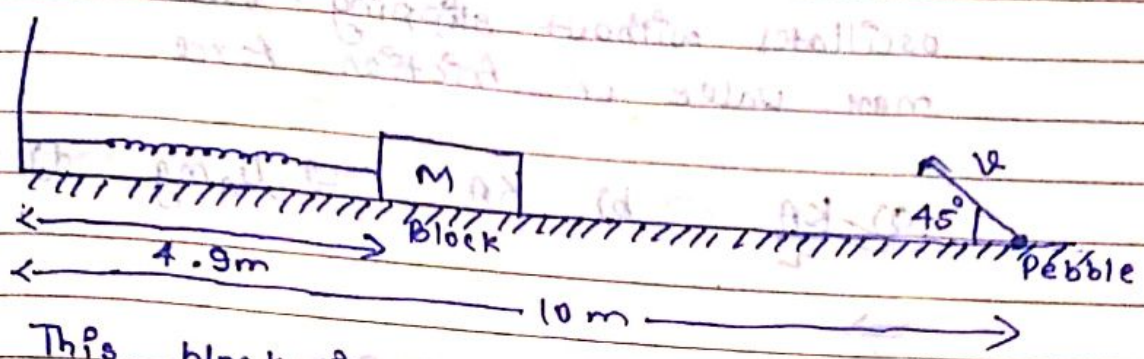
$$T = 2\pi \sqrt{\frac{m}{k \left(\frac{\sqrt{2} + 1}{\sqrt{2}} \right)}}$$~~



$$F_{net} = -\frac{3km}{2}$$

$$T = 2\pi \sqrt{\frac{m}{3k}}$$

IIT 2012:



This block is stretched to $x = 0.2 \text{ m}$ and released from rest at $t = 0$. Block \rightarrow SHM
 $\rightarrow \omega = \frac{\pi}{3}$. At $t = 0$ pebble is projected

If both meet at $t = 1 \text{ s}$, then find $v = ?$

- a) $\sqrt{50}$ b) $\sqrt{51}$ c) $\sqrt{52}$ d) $\sqrt{53}$

$\Rightarrow T = \frac{2\pi}{\omega} \times 3 = 6 \text{ sec}$

At $t = 1 \text{ sec}$,

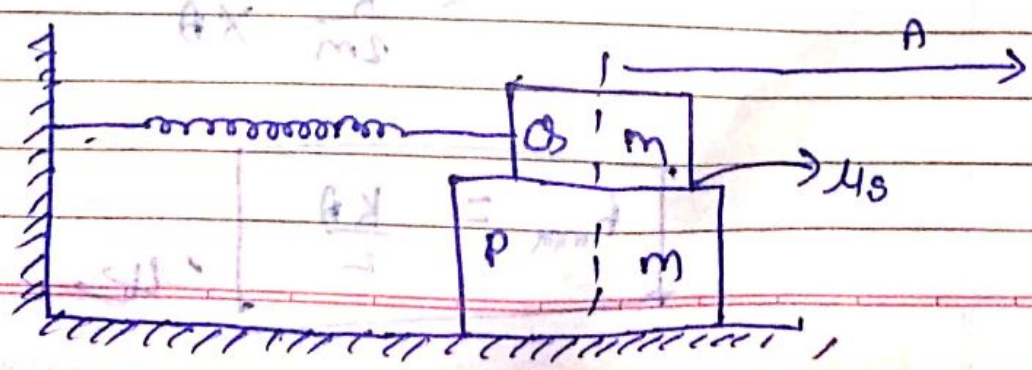
$R = 5 \text{ m}$

$= \frac{v^2}{g}$

$5 \times 10 = v^2$

$v = \sqrt{50} \text{ m/s}$

IIT 2004

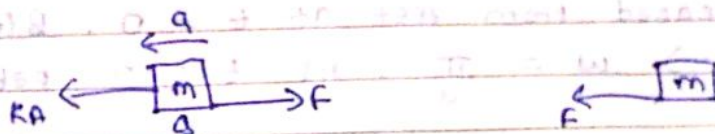


Two blocks pulled by distance A . m oscillates without slipping. What is the max value of friction force.

- a) $\frac{kA}{2}$ b) kA c) $\mu_s mg$ d) zero

⇒

At extreme points,



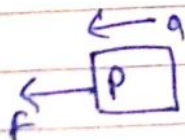
$$-F + kA = ma$$

$$F = ma$$

$$2F = kA$$

$$F = \frac{kA}{2}$$

OR



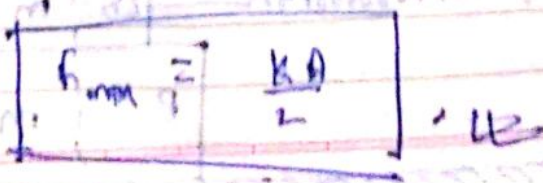
$$F = ma$$

$$F_{\max} = m a_{\max}$$

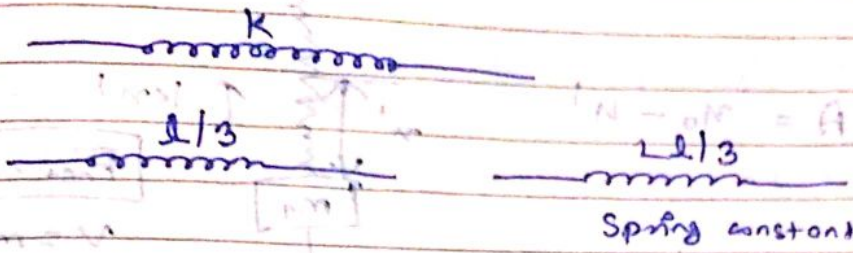
SHM,

$$a_{\max} = \omega^2 A$$

$$= \frac{k}{2m} \times A$$

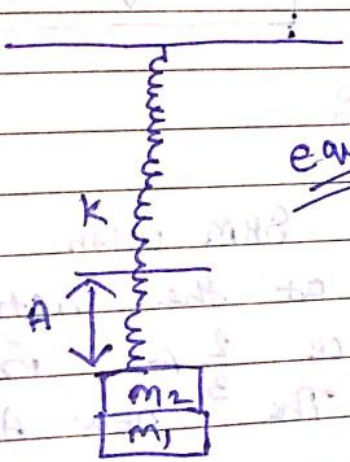


IIT 1999



⇒ $k_{eq} = \frac{3K}{2} \cdot \frac{2l}{3} = K \cdot l$

IIT 1981 :



equilibrium

m_1 is removed without any disturbance.

m_2 oscillates with freq ω & Amplitude A .
 $\omega = ?$, $A = ?$

⇒ $k_{eq} = (m_2 + m_1)g$

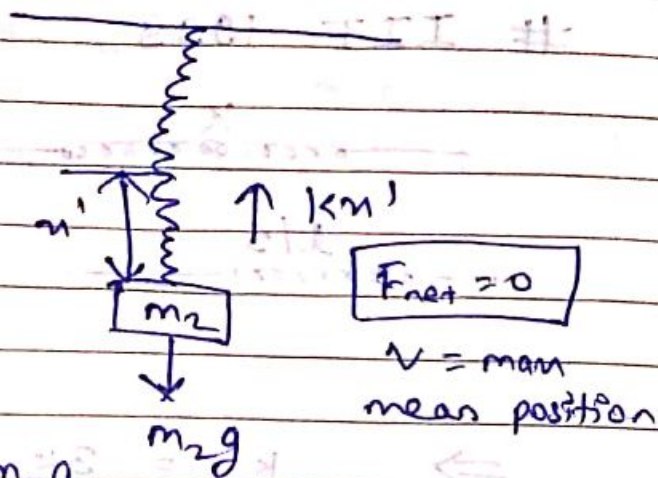
$x_{eq} = \frac{(m_1 + m_2)g}{K}$

~~$Kx - m_2g = m_2a$~~

~~$m_1g = m_2a$~~

~~$a = \frac{m_1g}{m_2}$~~

$$A = x_0 - x'$$



$$A = \frac{m_1 g + m_2 g}{k} - \frac{m_2 g}{k}$$

$$A = \frac{m_1 g}{k}$$

$$\omega = \sqrt{\frac{k}{m_2}}$$

JEE Mains 2016

A particle performs SHM with Amp 'A'. Its speed is trebled at the instant when it is at a distance $\frac{2}{3}A$ from equilibrium position. The new Amplitude of SHM \rightarrow

- a) $\frac{A\sqrt{41}}{3}$ b) $3A$ c) $A\sqrt{3}$ d) $\frac{7A}{3}$

$$\Rightarrow \frac{1}{2} m v^2 = \frac{1}{2} k x^2$$

$$\frac{1}{2} \times \cancel{m} \times \cancel{g} \times \left(\frac{5A^2}{g} \right) = \frac{1}{2} k x^2$$

$$\frac{5A^2}{2} = m x^2$$

$$v_f = 3v_p$$

$$\sqrt{A_f^2 - \left(\frac{2}{3}A\right)^2} = 3\sqrt{A^2 - \left(\frac{2}{3}A\right)^2}$$

$$A_f^2 - \frac{4A^2}{9} = \frac{8 \times 5A^2}{9}$$

$$A_f^2 = 5A^2 + \frac{4}{9}A^2$$

$$A_f = \frac{7A}{3}$$

IIT 2010

$V(x) = kx^2$ SHM, time period $\propto \sqrt{\frac{m}{k}}$

$$V(x) = \alpha x^4$$

dimensional analysis

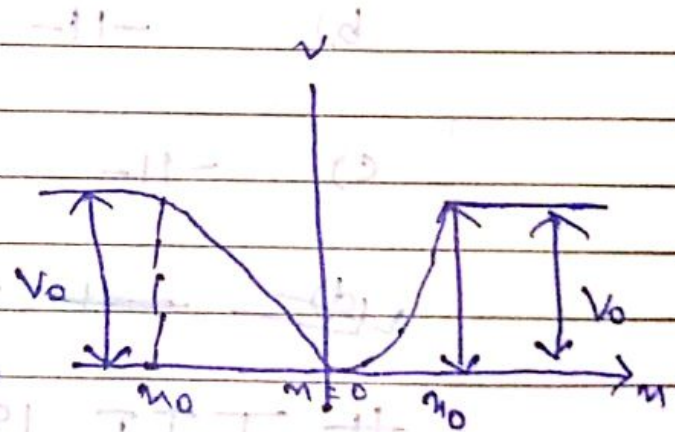


increases on both sides

of $x=0$

Periodic motion

$$V(x) = \alpha x^4 \quad |x| < x_0$$



$$V(x) = V_0 \text{ (constant)} \quad |x| > x_0$$

Total Energy \rightarrow infinity escape x

1) If the motion is periodic & Total Energy of particle is E , then

a) $E < 0$ ~~b) $E > 0$~~ ~~c) $V_0 > E > 0$~~

d) $E > V_0$

② Periodic, time period proportional \propto

a) $A \sqrt{\frac{m}{\alpha}}$ ~~b) $\frac{1}{A} \sqrt{\frac{m}{\alpha}}$~~ c) $A \sqrt{\frac{\alpha}{m}}$

d) $\frac{1}{A} \sqrt{\frac{\alpha}{m}}$

③ acceleration of this particle for

$|x| > X_0$

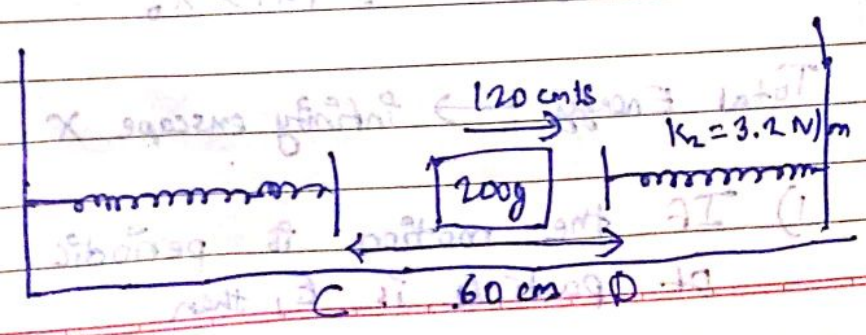
a) proportional to V_0

b) $-11 - \frac{V_0}{mX_0}$

c) $-11 - \sqrt{\frac{V_0}{mX_0}}$

~~d) zero~~

IIT 1985



if mass of 200g moves with 120 cm/sec between C & D. Find the time period of oscillation of 100 g mass.

- a) 2.42 s b) 2.82 s c) 3.92 s d) 3.62 s

$$\Rightarrow \frac{1}{2} \times 0.2 v^2 = 3.2 \times \pi^2$$

$$2 \times 0.12 \times 0.12 = \frac{16}{3.2 \times \pi^2}$$

$$t = \frac{1}{2} \times \frac{0.2 \times 0.12 \times 0.12}{0.03 \times 3.2}$$

$$= \frac{2 \times 12 \times 0.12 \times 0.12}{2 \times 2 \times 32} \times 0.03$$

$$= 0.015 \text{ sec}$$

$$\frac{0.12}{4} = \pi$$

$$\pi = 0.03 \text{ m}$$

$$= 30 \text{ cm}$$

$$2.03 + \frac{0.12}{3} = \pi$$

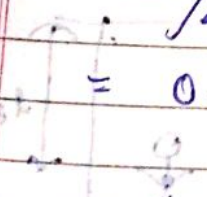
$$t = \frac{1}{2} \times \frac{0.2 \times 0.12 \times 0.12}{0.04 \times 1.8}$$

$$= \frac{0.12 \times 12 \times 3}{4 \times 18 \times 6}$$

$$= 0.02$$

$$\pi = 0.04 \text{ m}$$

$$= 40 \text{ cm}$$



$$T = \frac{120}{120} + \pi \sqrt{\frac{0.2}{3.2}} + \pi \sqrt{\frac{0.2}{1.8}}$$

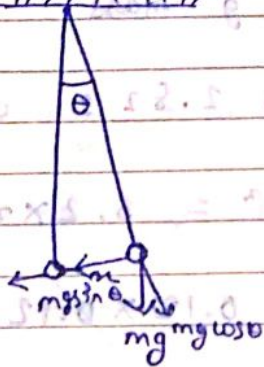
$$= 1 + \pi/4 + \pi/3$$

$$= 1 + 0.78 + 1.04$$

$$= 2.82 \text{ Sec.}$$

Time Period of Simple Pendulum :

if θ is small, then path is a straight line \rightarrow SHM



$$\theta = \frac{x}{l}$$

$$\sin \theta \approx \theta \approx \frac{x}{l}$$

$$F = -mg \sin \theta$$

$$K = \frac{mg}{l}$$

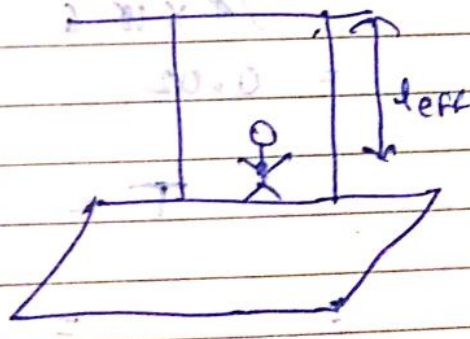
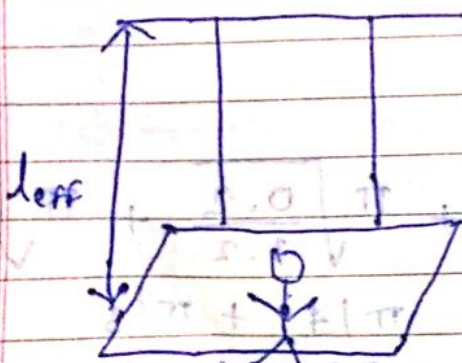
$$= -\frac{mgx}{l}$$

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$-kx = -\frac{mgx}{l}$$

$$= 2\pi \sqrt{\frac{ml}{mg}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

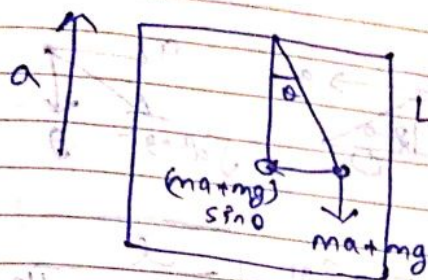


$$T = 2\pi \sqrt{\frac{l_{eff}}{g}}$$

$$\omega = \frac{2\pi}{T}$$

$$T = 2\pi \sqrt{\frac{l}{g_{\text{eff}}}}$$

Date _____
Page _____



Lift $F = -\frac{(ma + mg)l}{l}$
 $a_{\text{net}} = \frac{ma + mg}{l}$

$$T = 2\pi \sqrt{\frac{l}{a+g}}$$

IIT JEE 2005

Pendulum T_1
 Point of suspension move upward
 $y = kt^2$ ($k = 1 \text{ m/s}^2$) ($t \rightarrow \text{time}$)

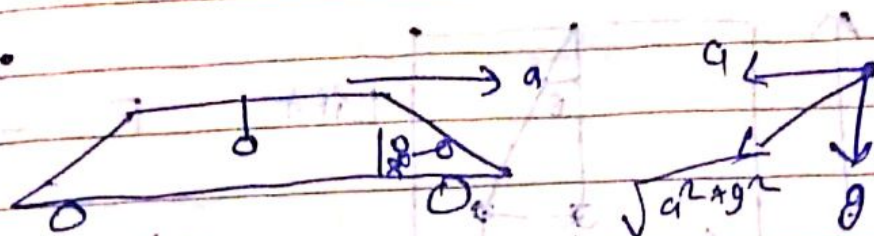
new Time Period $\rightarrow T_2$

$$\frac{T_1^2}{T_2^2} = \dots$$

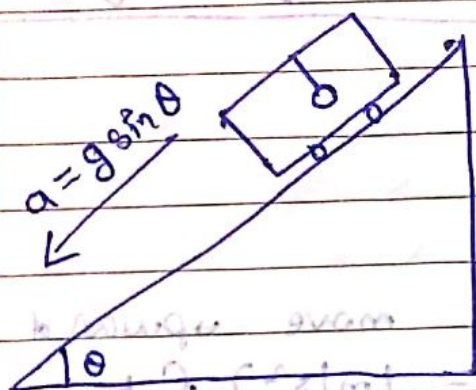
- a) $\frac{5}{6}$ b) $\frac{6}{5}$ c) 1 d) $\frac{1}{2}$

$\Rightarrow y = kt^2$
 $a = 2 \text{ sec}$

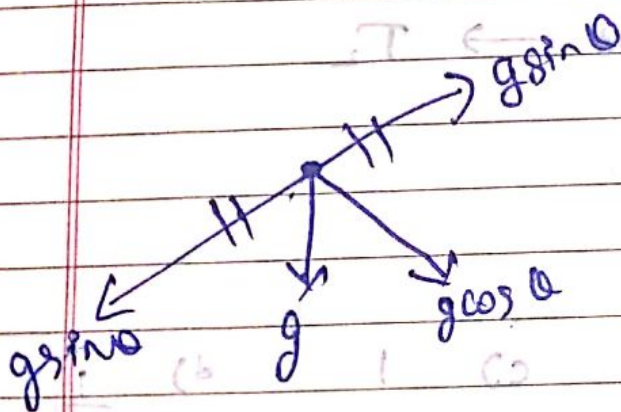
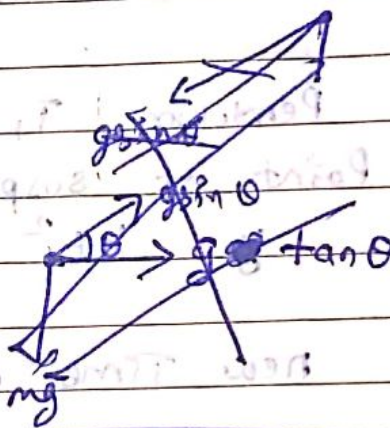
$$\frac{T_1^2}{T_2^2} = \frac{12}{10} = \frac{6}{5}$$



$$T = 2\pi \sqrt{\frac{l}{(a^2 + g^2)^{1/2}}}$$



$T = ?$



$$T = 2\pi \sqrt{\frac{l}{g \cos \theta}}$$

* Physical Pendulum

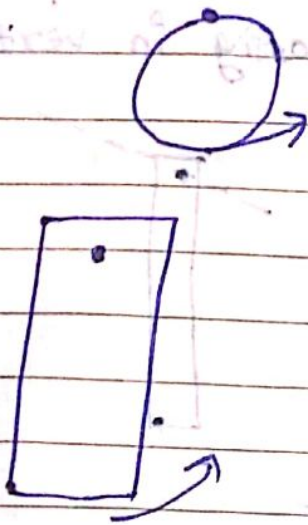
Linear SHM	Angular SHM
① $F = -kx$	$\tau = -k\theta$
② $T = \frac{2\pi}{\omega}$	$T = \frac{2\pi}{\omega}$

③ $\omega = \sqrt{\frac{k}{m l}}$

$\omega = \sqrt{\frac{k}{I}}$

④ $T = 2\pi \sqrt{\frac{m l}{k}}$

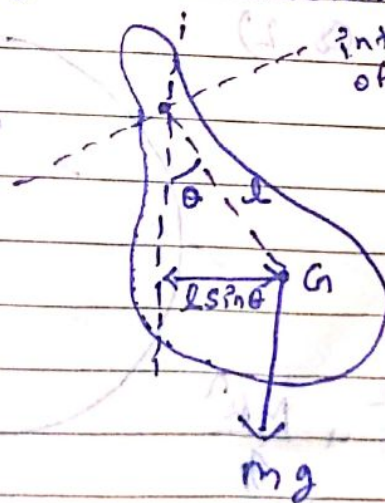
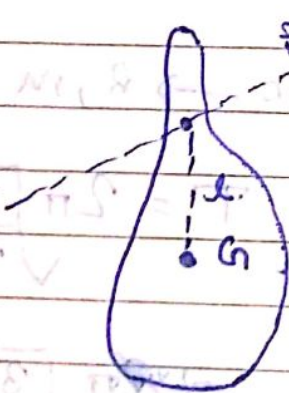
④ $T = 2\pi \sqrt{\frac{I}{k}}$



Rigid Body
(Pivoted) oscillate

Oscillations should be very very small of Any shape

Time Period of Physical Pendulum



Torque about axis of rotation

$$\begin{aligned} \tau &= mg(l \sin \theta) \\ &= -mg l \theta \end{aligned}$$

$$k = mg l$$

$$\tau = -k \theta$$

$$T = 2\pi \sqrt{\frac{I}{k}}$$

$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

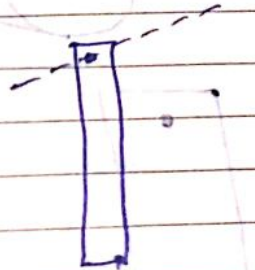
$I \rightarrow$ about axis of rotation

Q) Rod of Length L , Mass M , pivoted about one end & oscillating in vertical plane.

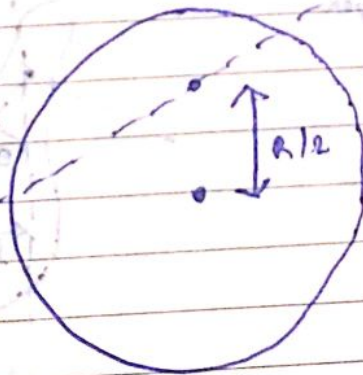
$T = ?$

$$T = 2\pi \sqrt{\frac{mL^2}{3mgL}}$$

$$T = 2\pi \sqrt{\frac{L}{3g}}$$



Q.2)



Disc $\rightarrow R, M$

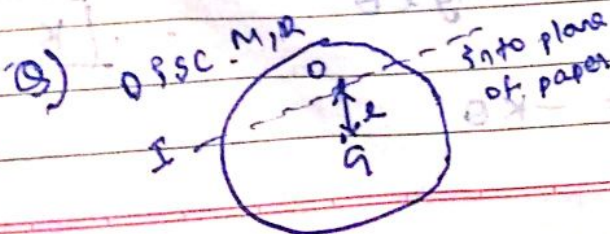
$$T = 2\pi \sqrt{\frac{I}{mgl}}$$

$$= 2\pi \sqrt{\frac{3mL^2 \times 2}{4mgL}}$$

$$T = 2\pi \sqrt{\frac{3L}{2g}}$$

$$I = \frac{MR^2}{2} + \frac{MR^2}{4}$$

$$= \frac{3MR^2}{4}$$



- For what distance $OG = T$
- ① Time Period \rightarrow Max
 - ② Time Period \rightarrow Min

Also find these Two Time Periods,

$$\Rightarrow T = 2\pi \sqrt{\frac{I}{mgL}}$$

$$T = 2\pi \sqrt{\frac{MR^2 + mL^2}{mgL}}$$

$$y = \frac{R^2}{2gL} + \frac{L}{g}$$

$T \rightarrow$ min, $y \rightarrow$ min

$$\frac{dy}{dL} = 0$$

$$-\frac{R^2}{2gL^2} + \frac{1}{g} = 0$$

$$L^2 = \frac{R^2}{2}$$

$$L = \frac{R}{\sqrt{2}}$$

$$T = 2\pi \sqrt{\left(\frac{R^2}{2} + \frac{R^2}{2}\right) \frac{1}{g}}$$

$$T = 2\pi \sqrt{\frac{\sqrt{2}R}{g}}$$

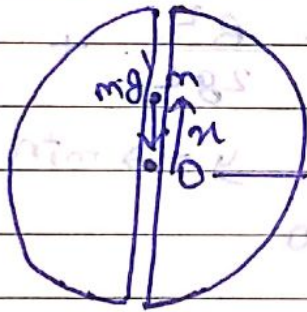
$$T = 2\pi \sqrt{\frac{1.414 R}{g}}$$

$T_{\text{max}} \longrightarrow \infty$

$$l = 0$$

- Tunnel through Earth

$$T = 2\pi \sqrt{\frac{R}{g}} \approx 84.6 \text{ min}$$



mean position $\int F_{\text{net}} = 0$
 $g = 0$

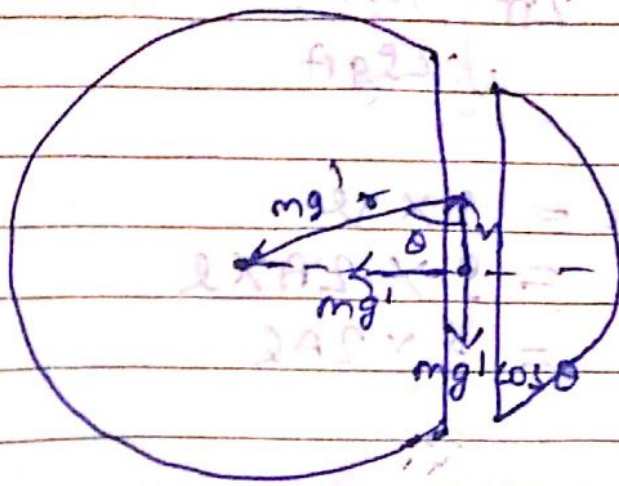
$$F_{\text{net}} = mg' = -\frac{mg r}{R}$$

$$k = \frac{mg}{R}$$

$$T = 2\pi \sqrt{\frac{m R}{mg}}$$

$$T = 2\pi \sqrt{\frac{R}{g}}$$

$$\approx 84.6 \text{ min}$$



Equilibrium position

$$g' = \frac{gR}{R}$$

$$F_{net} = \frac{mgx}{R} \cos \theta$$

$$= \frac{mgx}{R} \times \frac{x}{R}$$

$$= -\frac{mgx^2}{R}$$

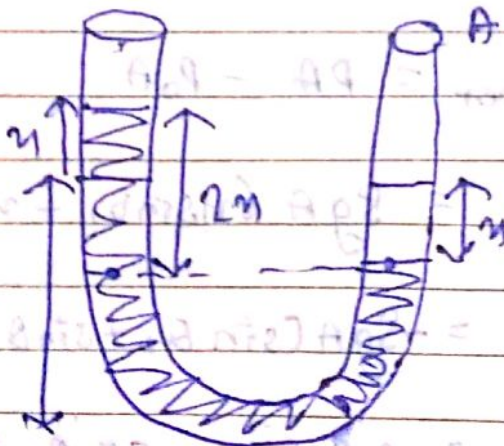
$$k = \frac{mg}{R}$$

$$-kx = -\frac{mgx}{R}$$

$$T = 2\pi \sqrt{\frac{R}{g}}$$

Oscillation of Liquid in a U-tube

(9)



$$F_{net} = PA - P_0A$$

$$F_{net} = (P_0 + \rho g 2n)A - P_0A$$

$$= -\rho g 2nA$$

$$F_{net} = -2\rho g A n$$

$$T = 2\pi \sqrt{\frac{m}{2SgA}}$$

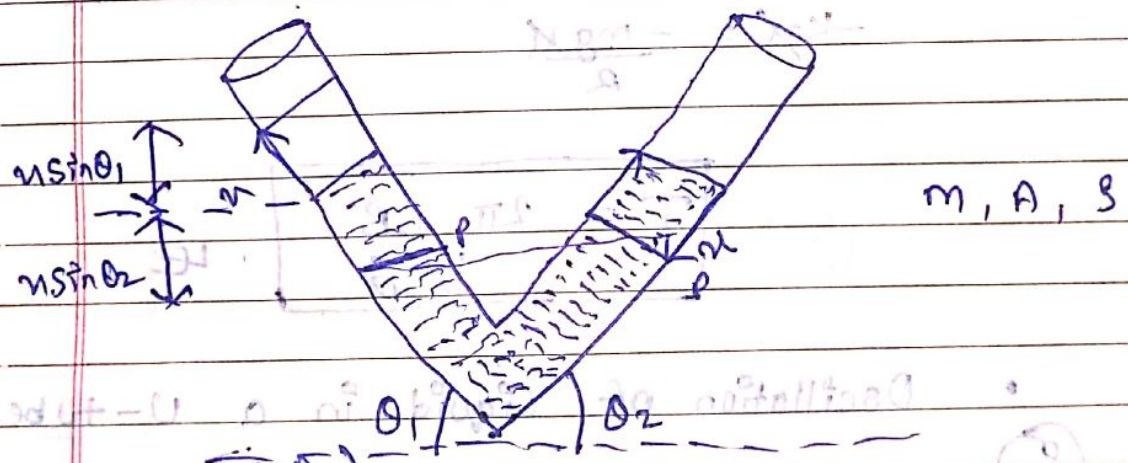
$$m = S \times l$$

$$= S \times 2A \times l$$

$$= S \times 2Al$$

$$T = 2\pi \sqrt{\frac{2SAl}{2SgA}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$



$$P = P_0 + Sg (ns \sin \theta_1 + ns \sin \theta_2)$$

$$F_{net} = PA - P_0A$$

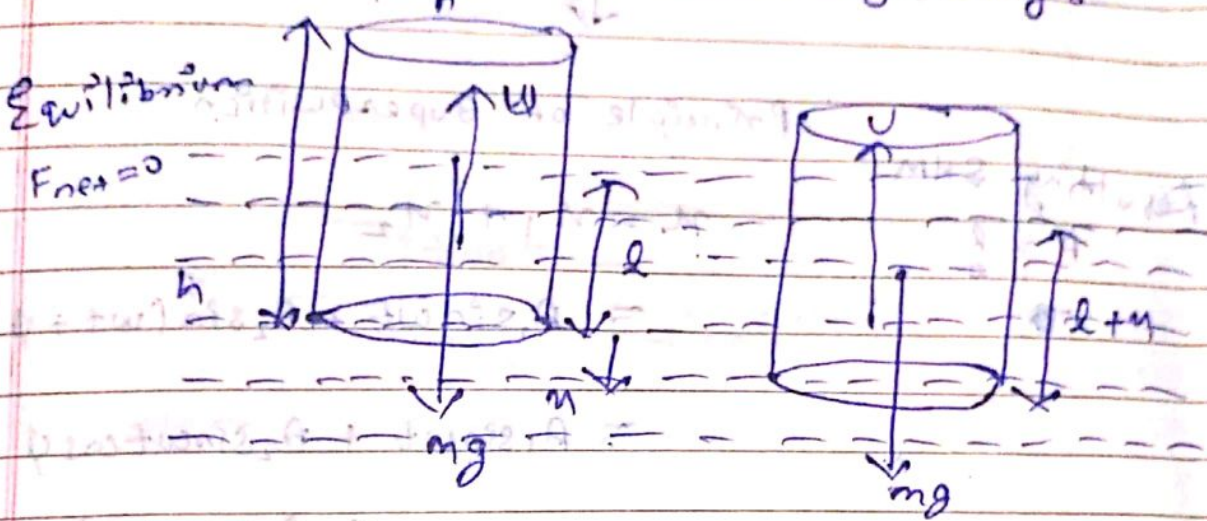
$$= SgA (ns \sin \theta_1 + ns \sin \theta_2)$$

$$= -SgA (s \sin \theta_1 + s \sin \theta_2) 2l$$

$$k = SgA (s \sin \theta_1 + s \sin \theta_2)$$

$$T = 2\pi \sqrt{\frac{m}{\rho g A (\cos\theta_1 + \sin\theta_2)}}$$

• Oscillation of a Floating Body :-



Equilibrium
 $F_{net} = 0$

$$U = mg$$

$$V \rho g = mg$$

$$A h \rho g = mg \quad \text{--- (1)}$$

$$F_{net} = U - mg$$

$$= V' \rho g - mg$$

$$= A(l+x) \rho g - mg$$

$$= A l \rho g + A x \rho g - mg$$

$$F_{net} = - A \rho g x$$

$$F_{net} = - k x$$

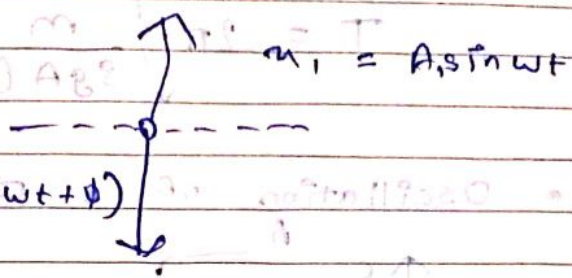
$$k = A \rho g$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$= 2\pi \sqrt{\frac{A l \rho}{A \rho g}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

Combination of SHM



Principle of Superposition

Resulting SHM

$A = ?$

$x = x_1 + x_2$

$= A_1 \sin wt + A_2 \sin (wt + \phi)$

$= A_1 \sin wt + A_2 \sin wt \cos \phi$

$+ A_2 \cos wt \sin \phi$

$= \sin wt (A_1 + A_2 \cos \phi)$

\downarrow
 $A \cos \theta$

$+ \cos wt (A_2 \sin \phi)$

\downarrow
 $A \sin \theta$

$x = A \sin wt \cos \theta + A \cos wt \sin \theta$

$x = A \sin (wt + \theta)$

$A \sin \theta = A_2 \sin \phi$ — (1)

$A \cos \theta = A_1 + A_2 \cos \phi$ — (2)

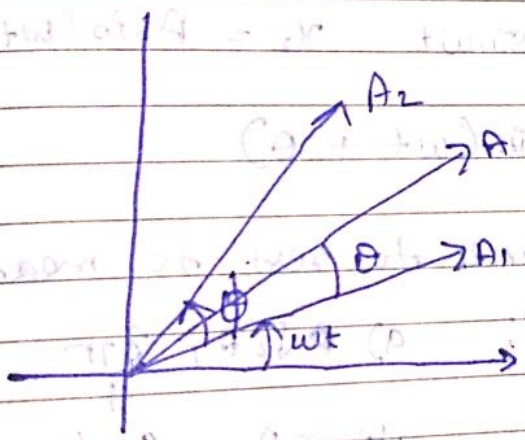
$$A^2 = A_2^2 \sin^2 \phi + A_1^2 + A_2^2 \cos^2 \phi + 2A_1 A_2 \cos \phi$$

$$A^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \phi$$

Divide (i) + (ii)

$$\tan \theta = \frac{A_2 \sin \phi}{A_1 + A_2 \cos \phi}$$

Shortcut :-



$$x_1 = A_1 \sin \omega t$$

$$x_2 = A_2 \sin(\omega t + \phi)$$

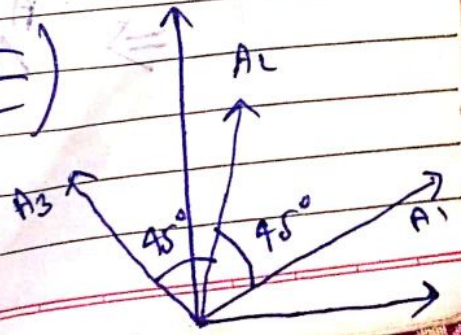
$$x = A \sin(\omega t + \theta)$$

(i) $x_1 = 2 \sin \omega t$

(ii) $x_2 = 2\sqrt{2} \sin(\omega t + \frac{\pi}{4})$

(iii) $x_3 = \sin(\omega t + \frac{\pi}{2})$

$\Rightarrow A_x = 2 + 2 = 4$
 $A_y = 1 + 2 = 3$



$$A = \sqrt{A_x^2 + A_y^2}$$

$$= 5$$

$$\tan \theta = \frac{3}{4}$$

$$\theta = 37^\circ$$

$$x = 5 \sin(\omega t + 37^\circ)$$

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$$x_1 = A \sin \omega t \quad x_2 = A \sin(\omega t + \frac{2\pi}{3})$$

$$x_3 = B \sin(\omega t + \theta)$$

particle comes to rest to mean position.

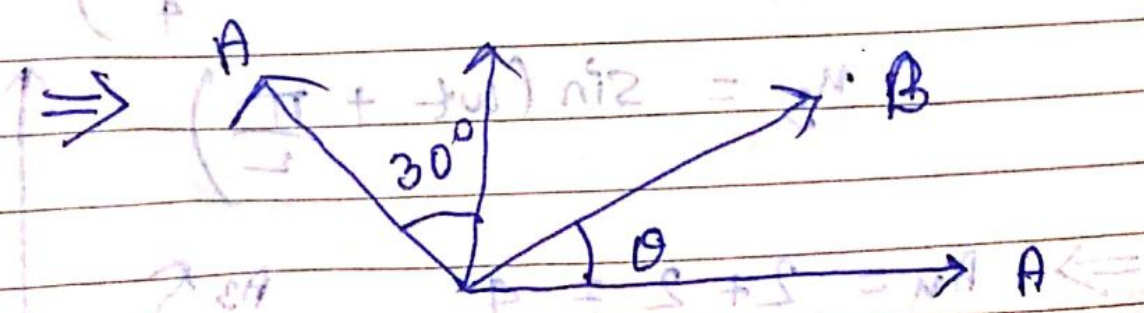
$$B, \theta = ?$$

a) $\sqrt{2}A, \frac{3\pi}{4}$

~~b) $A, \frac{4\pi}{3}$~~

c) $\sqrt{3}A, \frac{5\pi}{6}$

~~d) $A, \frac{\pi}{3}$~~



$$A + B \cos \theta = \frac{A}{2} = 0$$

$$B \cos \theta = -\frac{A}{2}$$

$$\frac{\sqrt{3}A}{2} + B \sin \theta = 0$$

$$B \sin \theta = -\frac{\sqrt{3}A}{2}$$

$$\tan \theta = -\sqrt{3}$$

$$\boxed{\theta = 60^\circ}$$
 But, \because B is opp.

$$\boxed{\theta = 240^\circ}$$

$$\boxed{B = A}$$